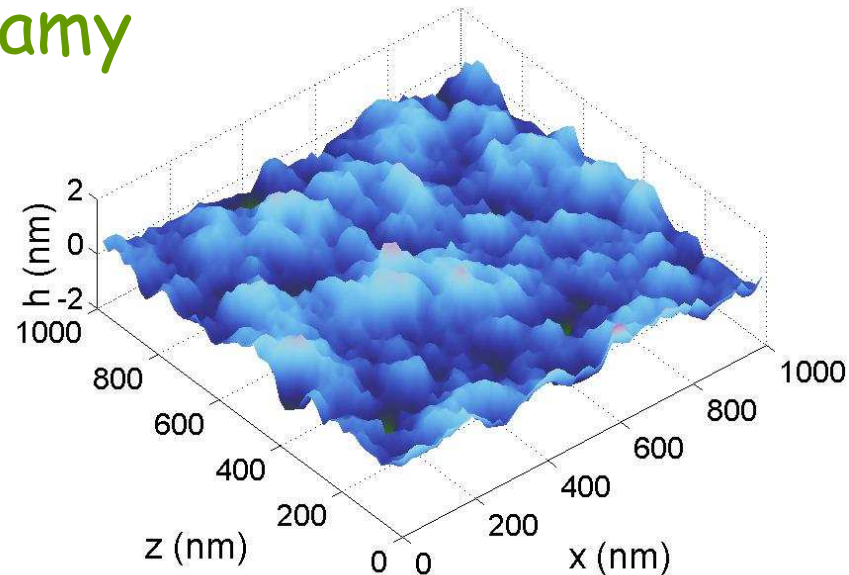
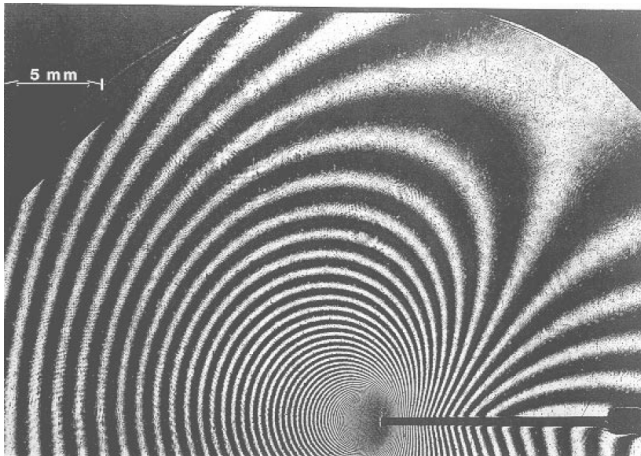


Rupture et endommagement des matériaux vitreux

Daniel Bonamy



CEA Saclay, DSM/IRAMIS/SPCSI, 91191 Gif sur Yvette

GDR Matériaux Vitreux, Dourdan, 10 septembre 2009



PLAN DE L'EXPOSÉ

Mécanique de la rupture: Théorie **continue**

Rôle des défauts, failles de Griffith

Propagation de fissures: critère de stabilité,
équation de mouvement, trajectoire...

Rôle du désordre microstructural: Description **stochastique**

Fissuration intermittente

ténacité effective

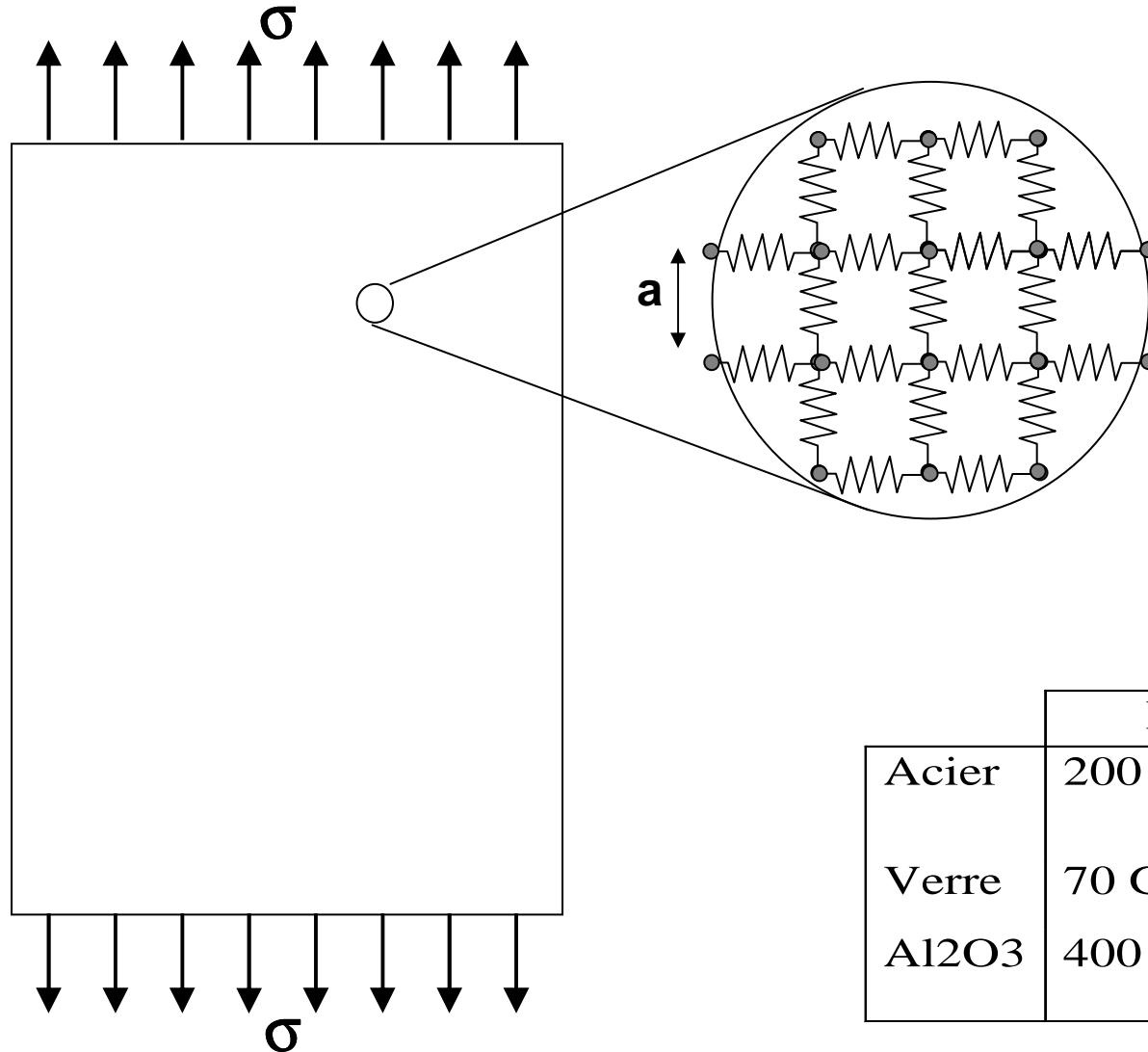
Faciès de rupture

Instabilité dynamique, branchement



Contrainte à rupture d'un solide « parfait »

Vue microscopique



E module d' Young
 a distance entre atomes
 σ_F contrainte de rupture

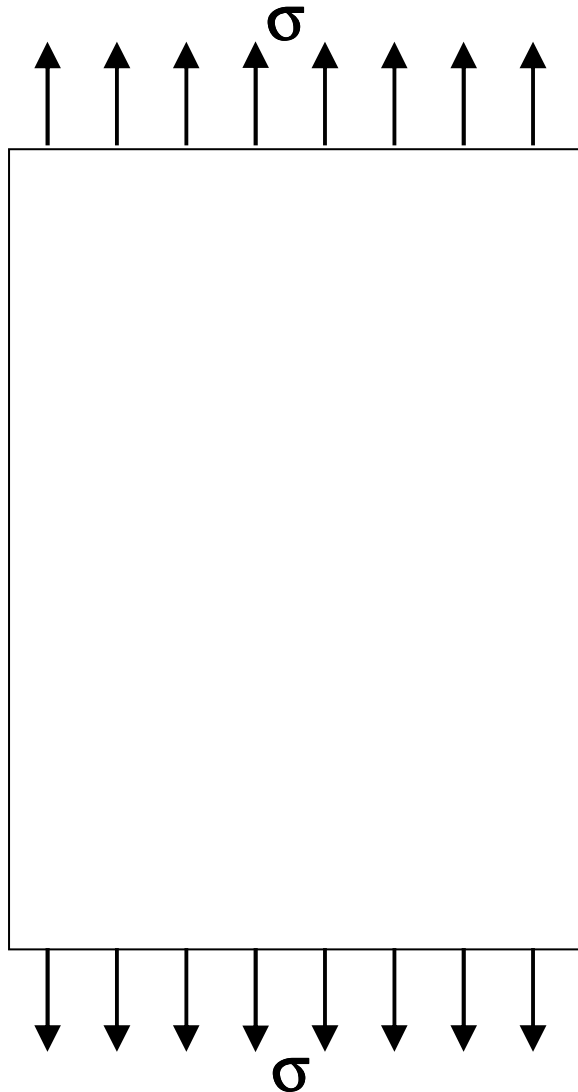
Rupture : $\sigma_F \sim E$

	E	σ_f
Acier	200 GPa	100 Mpa 2 GPa
Verre	70 GPa	300 MPa
Al ₂ O ₃	400 GPa	<100 Mpa

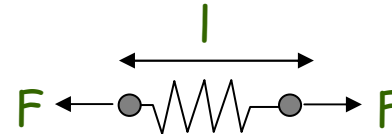
???



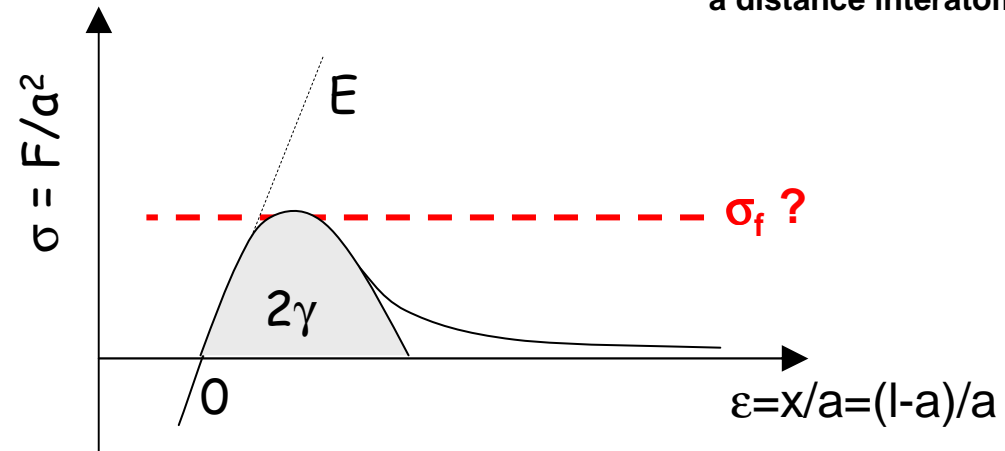
Contrainte à rupture d'un solide « parfait »



Vue microscopique



E module d'Young
 γ énergie de surface
a distance interatomique



Rupture : $\sigma_f \approx \sqrt{E\gamma/a}$

VERRE :

???

$a \sim 1 \text{ \AA}$
 $E \sim 70 \text{ GPa}$
 $\gamma \sim 0.1 \text{ J/m}^2$

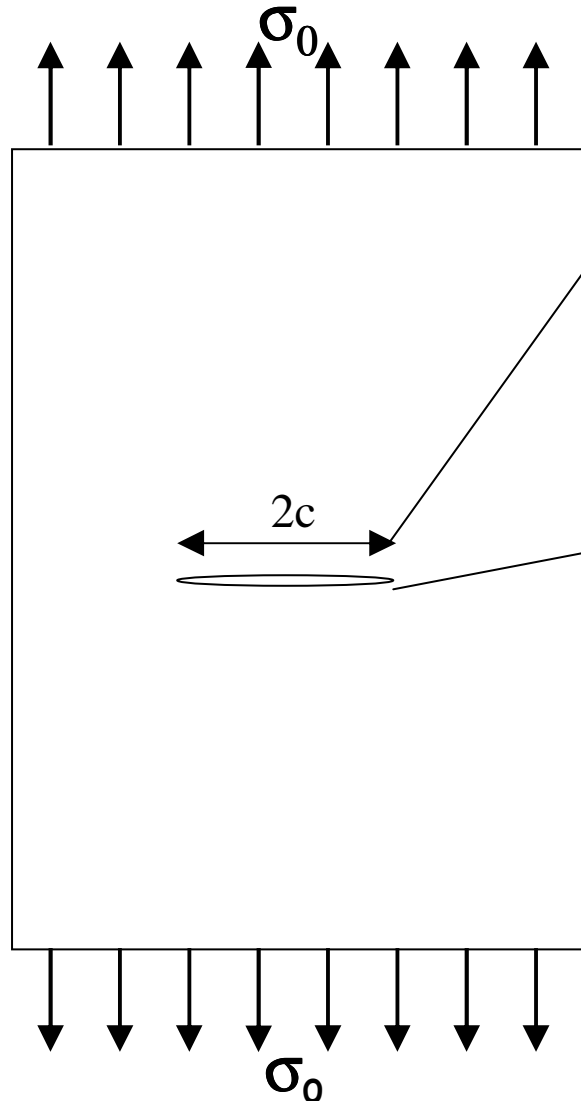


$\sqrt{E\gamma/a} \sim 8 \text{ GPa}$

$\sigma_f \sim 300 \text{ MPa}$

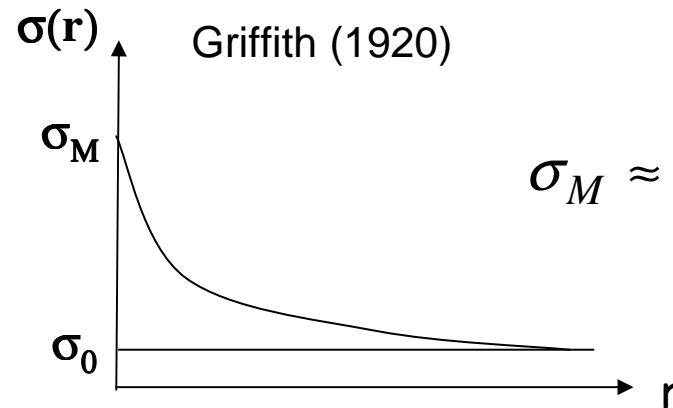


Contrainte à rupture d'un solide: Rôle des défauts



Concentration des contraintes

Griffith (1920)



$$\sigma_M \approx 2\sigma_0 \sqrt{c/\rho}$$

Rupture : $\sigma_f \approx \sqrt{E\gamma/a} / (2\sqrt{c/\rho})$

VERRE :

$a \sim 1 \text{ \AA}$
 $E \sim 70 \text{ GPa}$
 $\gamma \sim 0.1 \text{ J/m}^2$
 $\rho \sim a$
 $c \sim 100 \text{ nm}$

prédiction
 $\sigma_f \sim 500 \text{ MPa}$

expérience
 $\sigma_f \sim 300 \text{ MPa}$



Distribution de contraintes à rupture

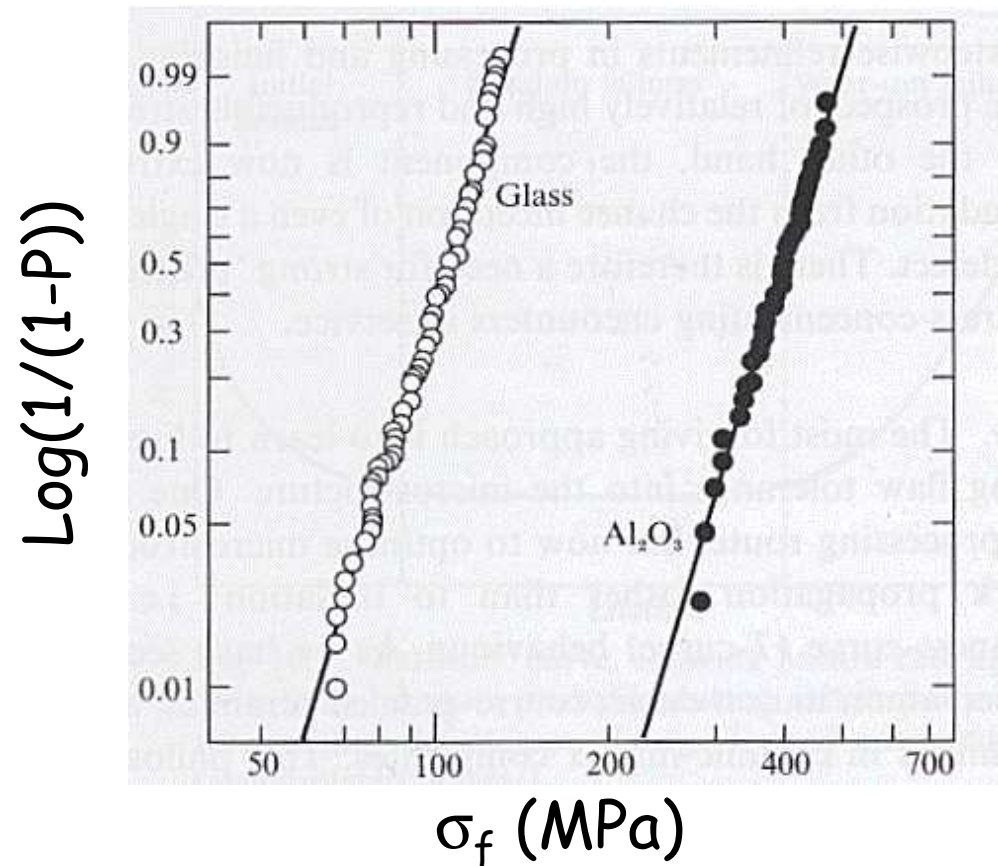
... Dominé par les liens les plus faibles

Statistique d'événements extrêmes

Loi de Weibull

$$P_{<} = 1 - \exp(-(\sigma_f / \sigma_*)^m)$$

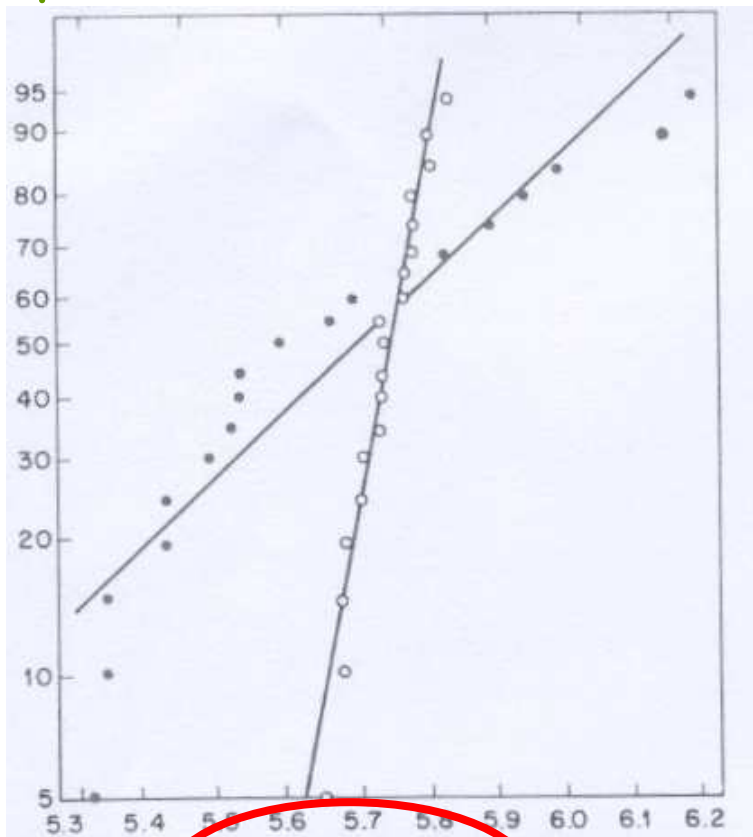
Ex: verre sodocalcique et alumine (d'après Lawn, 1993)





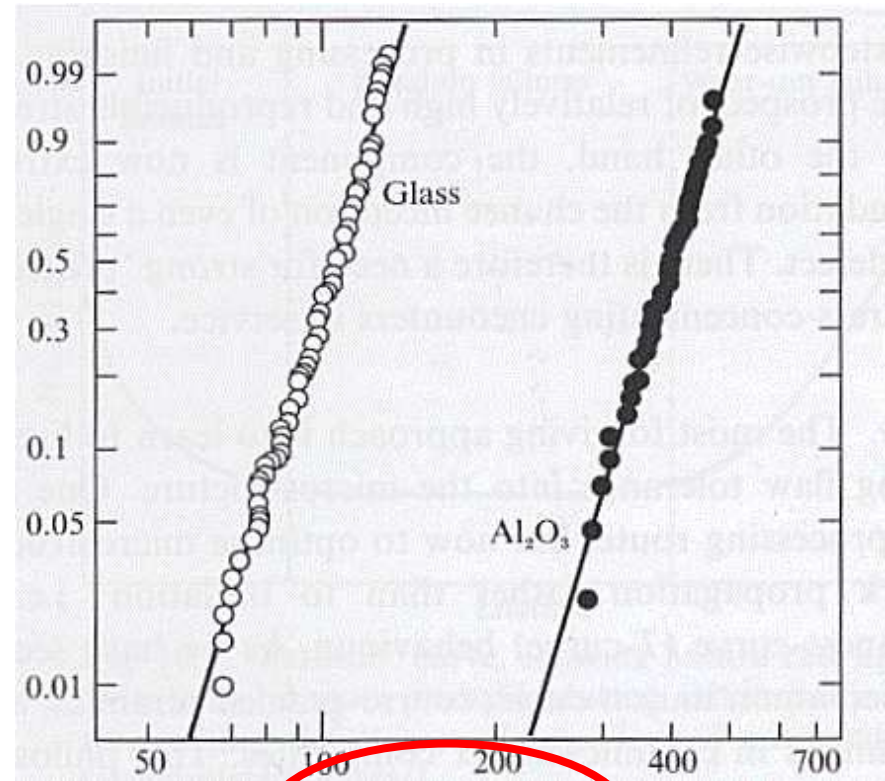
Distribution de contraintes à rupture

Fibres de verre, $d=100\mu\text{m}$,
« parfaites » (d'après Kurkjian, 1982)



σ_f (GPa)

Ex: verre sodocalcique
et alumine (d'après Lawn, 1993)

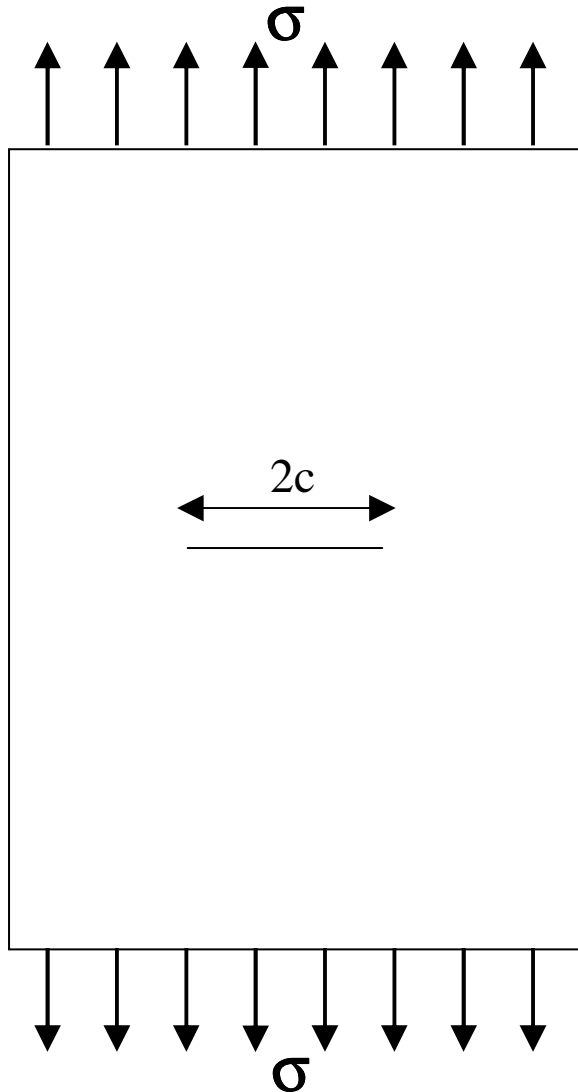


σ_f (MPa)

... Et effets d'échelles

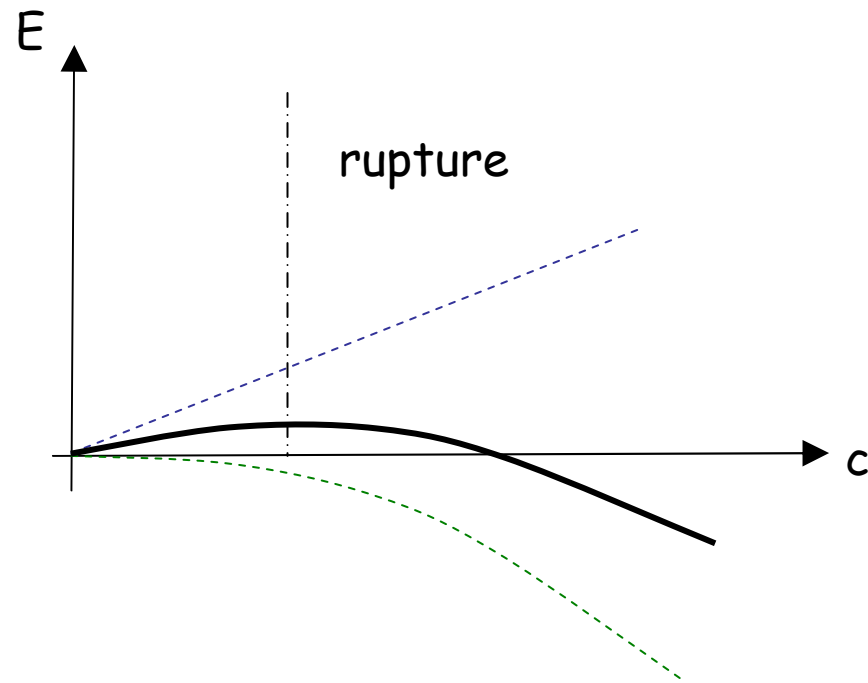


Théorie de Griffith: approche thermodynamique



Energie total Energie mécanique Energie de surface

$$\begin{aligned} E &= E_M + E_s \\ E &= -\pi c^2 \sigma^2 / E + 4 \gamma c \end{aligned}$$

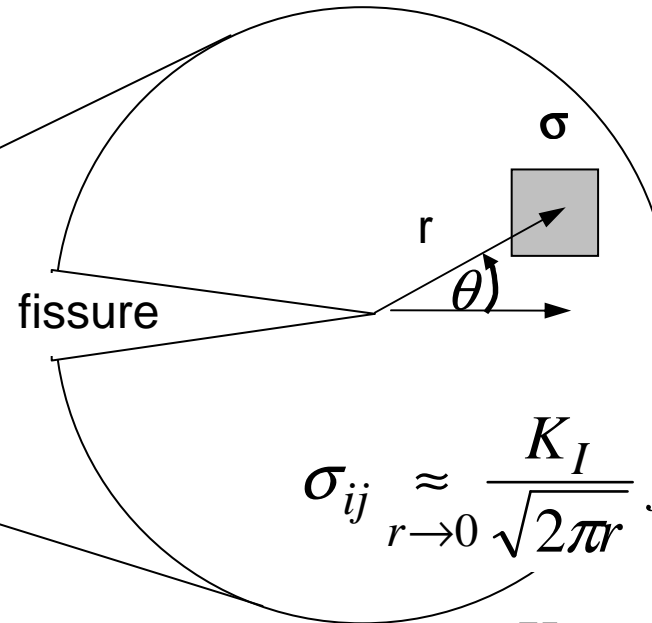
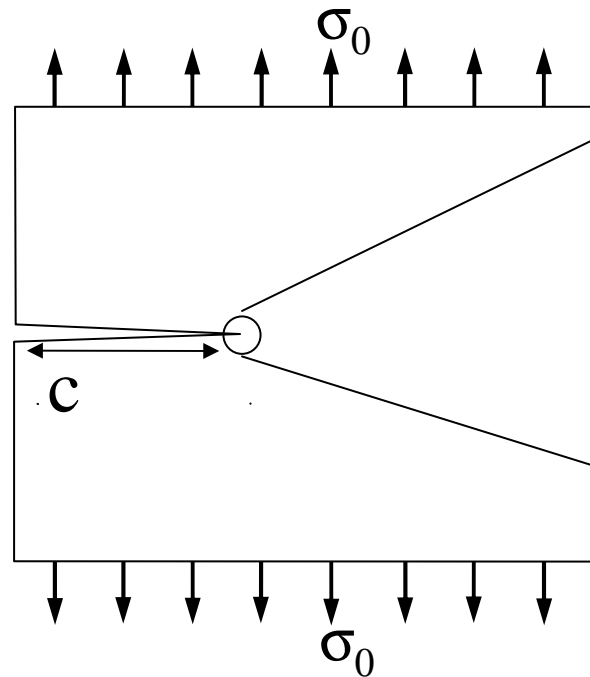


Taux d'énergie mécanique relâchée

Critère de rupture: $G = -dE_M/dc > 2\gamma$



Mécanique Linéaire Elastique de la Rupture (MLER)



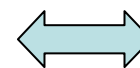
*Irwin (1955),
Orrowan (1957)*

$$\sigma_{ij} \underset{r \rightarrow 0}{\approx} \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta)$$

$$K_I = \sigma_0 \sqrt{c} F(\text{géométrie})$$

$K_I > K_{Ic}$: fissure instable

K_I facteur d'intensité des contraintes
 K_{Ic} ténacité



$G > \Gamma$ ($\sim 2\gamma$)

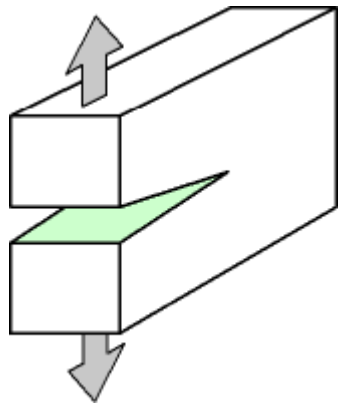
$$G = K_I^2 / E$$

$$\Gamma = K_{Ic}^2 / E$$

Γ énergie de fracture

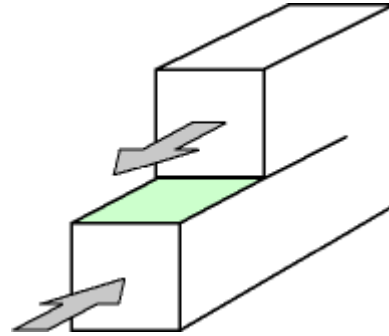


Modes de rupture



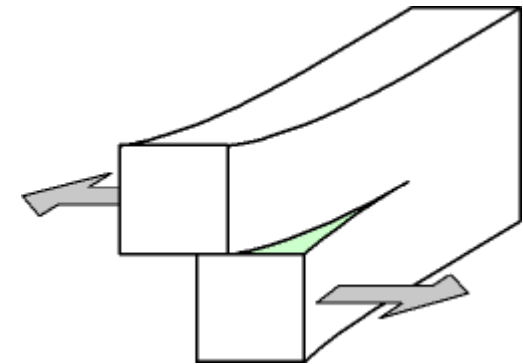
Mode I
Tension, ouverture

K_I



Mode II
Cisaillement dans
le plan, glissement

K_{II}

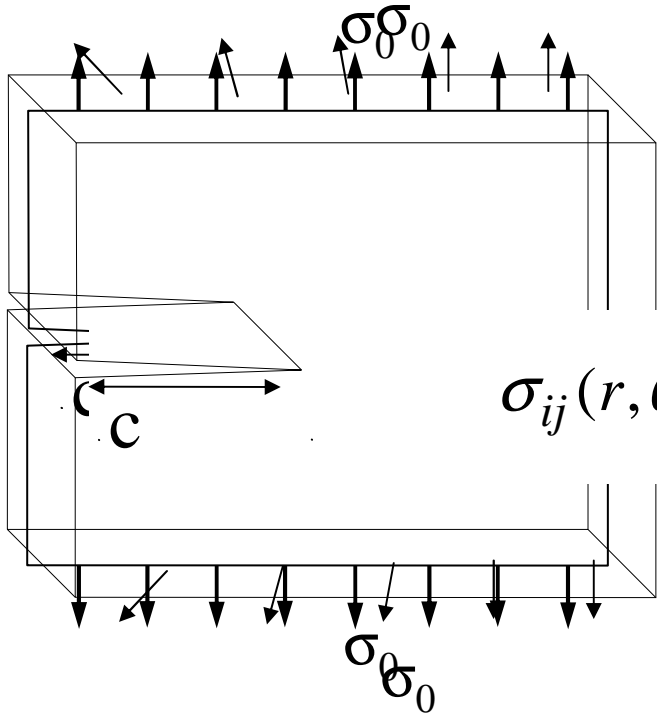


Mode III
Cisaillement hors plan
déchirement

K_{III}



Mécanique Linéaire Elastique de la Rupture (MLER)



*Irwin (1955),
Orrowan (1957)*

$$\sigma_{ij}(r, \theta) \underset{r \rightarrow 0}{\approx} \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} g_{ij}(\theta) + \frac{K_{III}}{\sqrt{2\pi r}} h_{ij}(\theta)$$

$$G = K_I^2 / E + K_{II}^2 / E + (1 + \nu) K_{III}^2 / E$$

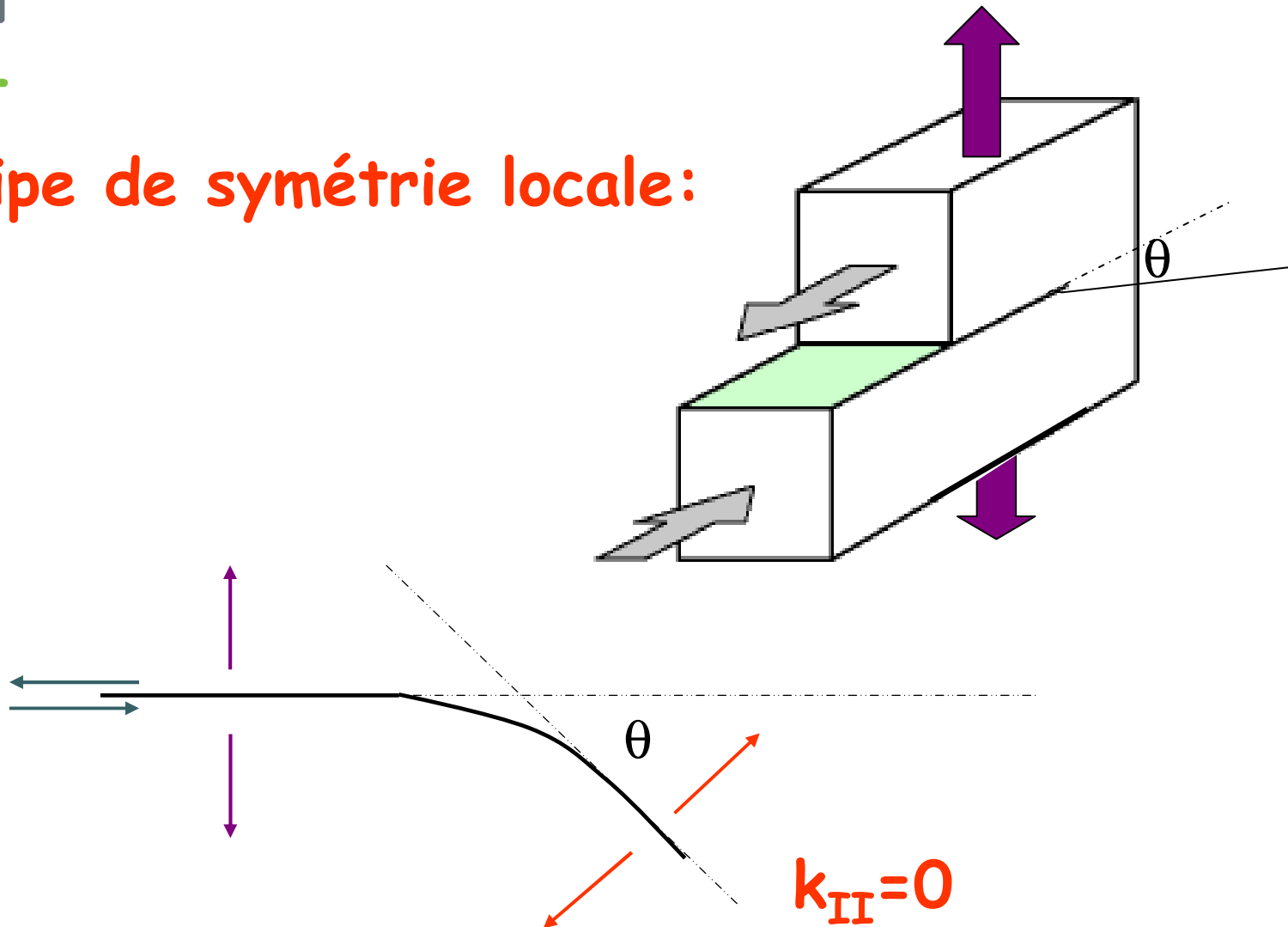
$$G > \Gamma$$

~~$$G = K_I^2 / E$$~~



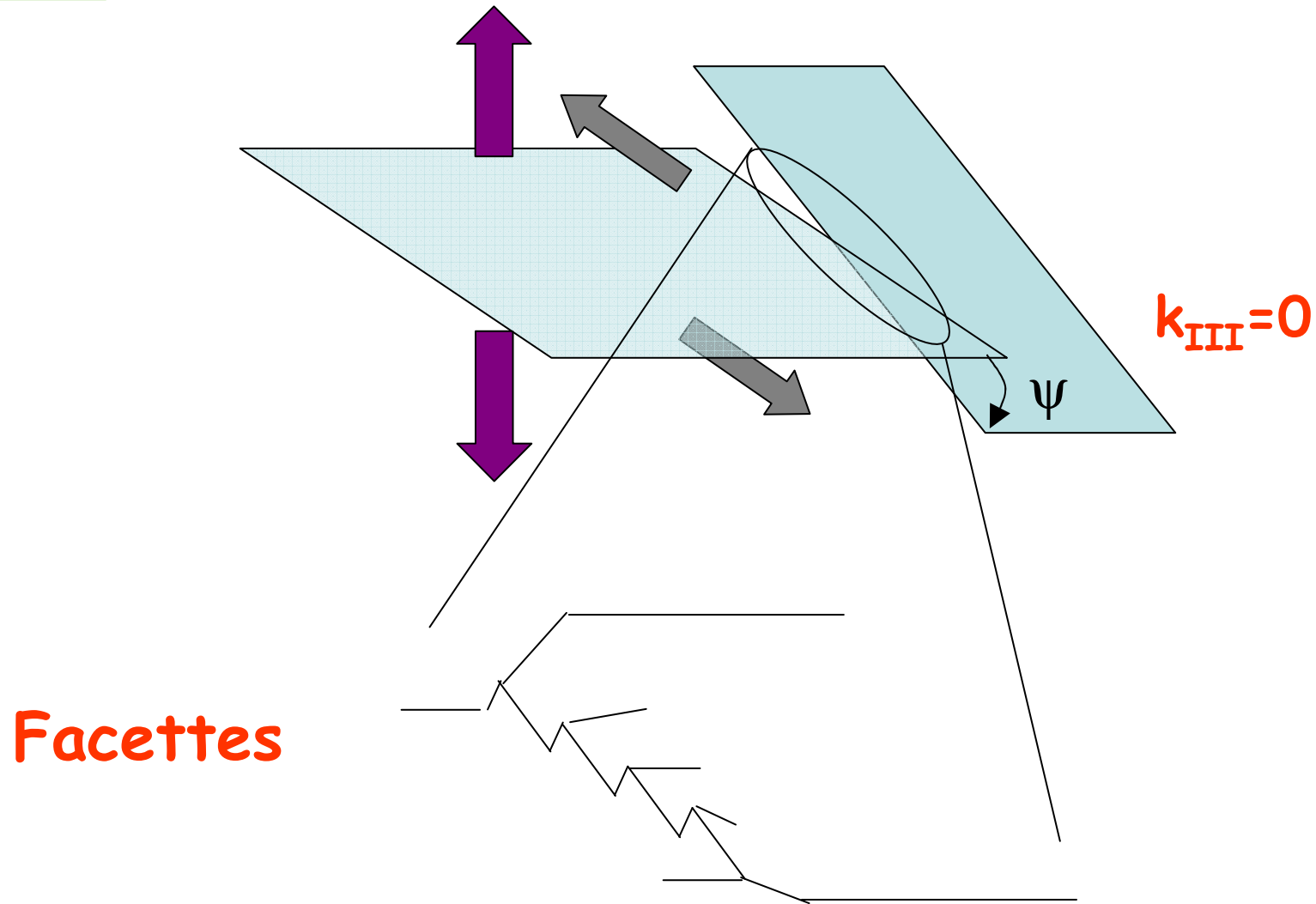
Critère de trajectoire: mode I/II

Principe de symétrie locale:





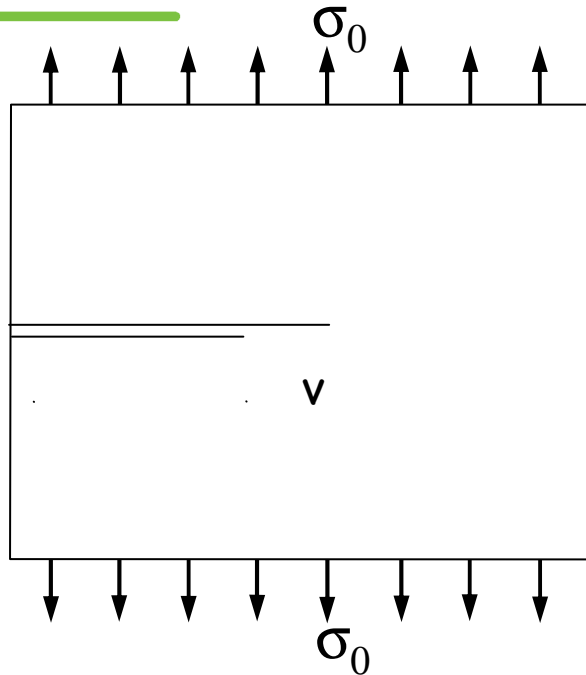
Critère de trajectoire: mode I/III





Equation de mouvement

Freund (1990),



Calculs élastodynamiques

$$\sigma_{ij} \underset{r \rightarrow 0}{\approx} \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta, \nu)$$

$$G = A(\nu) K_I^2 / E$$

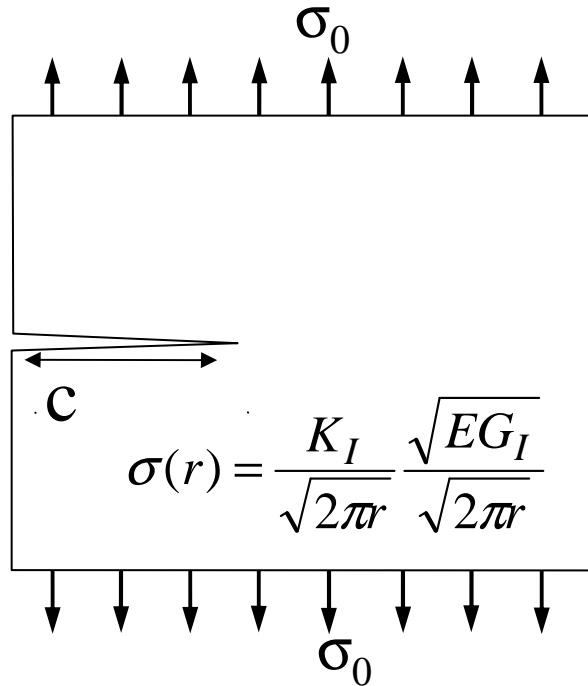
$$\text{avec } A(\nu) \sim (1 - \nu / C_R)$$

C_R vitesse de Rayleigh (ondes sonores sur une surface)

Vitesse donnée par: $A(\nu) K_I^2 / E = \Gamma$



Mécanique Linéaire Elastique de la Rupture: Bilan



Equation de trajectoire:

Cotterell & Rice (1980)

Plan de fracture choisi de manière à maximiser la contrainte de tension

$$K_{II} = 0$$

Critère de rupture:

$$G_I > \Gamma \quad (\sim 2\gamma)$$

ou $K_I > K_{Ic}$

Equation de mouvement:

Freund (1990)

$$A(v) K_I^2 / E = \Gamma$$

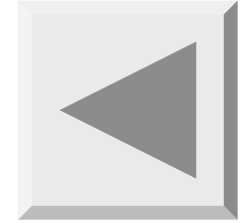
Facteur relativiste :

$$A(V) \sim (1 - V/C_R)$$

C_R vitesse de Rayleigh



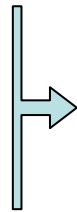
Critère de rupture: prédictions



➤ Verre de Silice

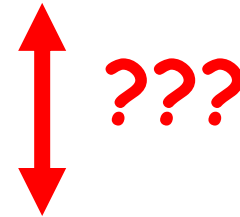
$$K_{Ic} \sim 0.8 \text{MPa}\cdot\text{m}^{1/2}$$

$$E \sim 70 \text{GPa}$$



Énergie de fracture

$$\Gamma \sim 10 \text{ J/m}^2$$



Énergie de surface
« atomique »

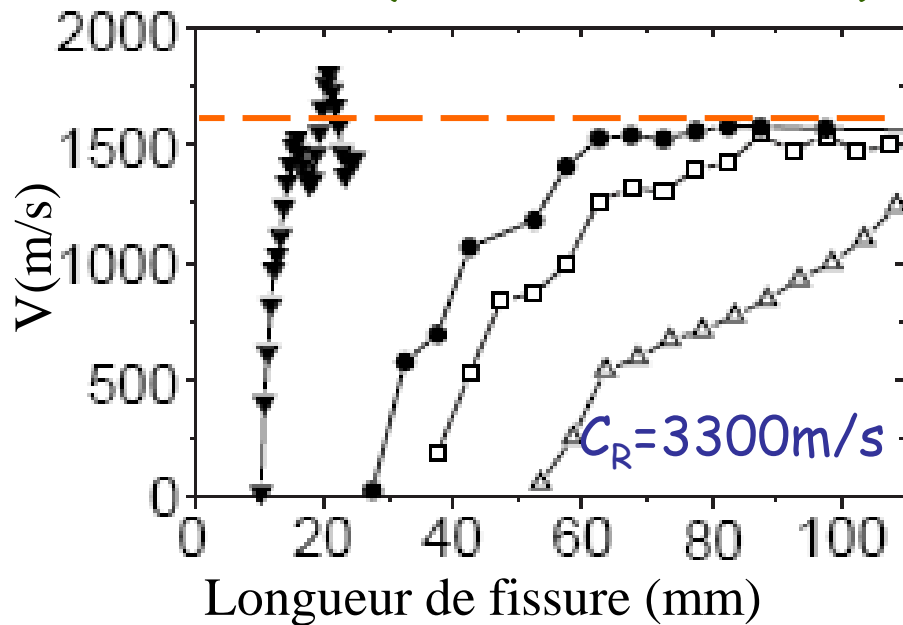
$$2\gamma \sim 0.2 \text{ J/m}^2$$



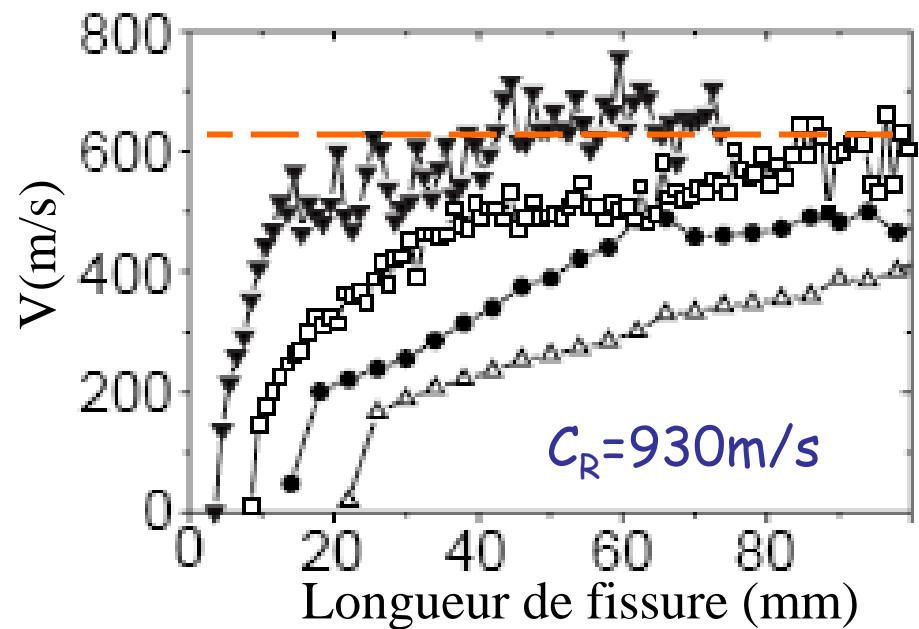
Equation de mouvement: prédictions

➤ Vitesse limite théorique = vitesse de Rayleigh

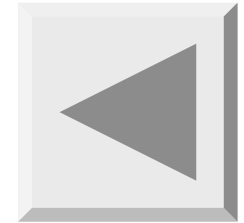
Verre (Sharon et al, 1999)



Plexiglas (Sharon et al, 1999)



Expériences: $V_{\max} \leftarrow \sim 0.6 C_R$!!!



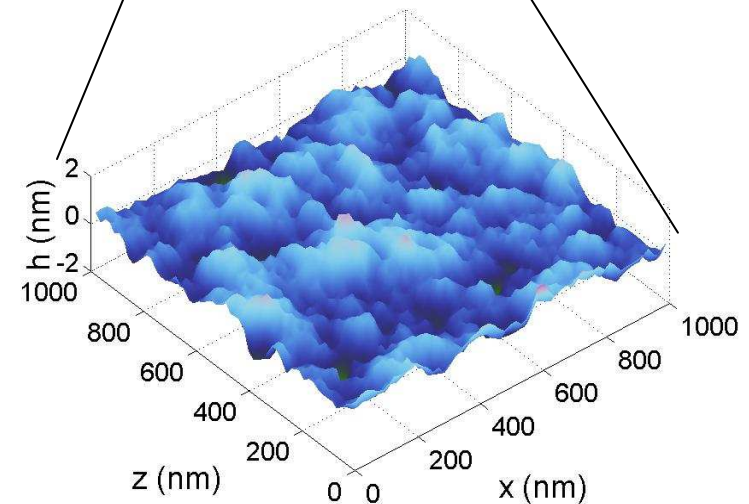
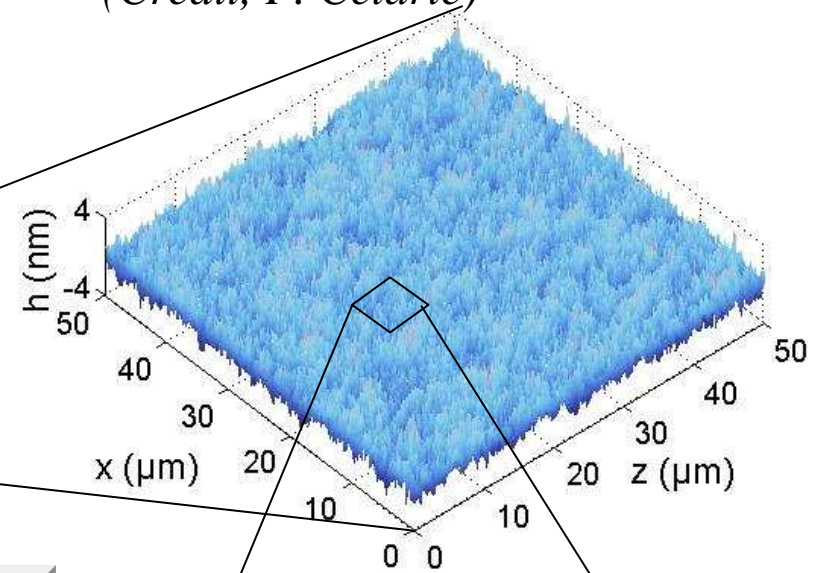
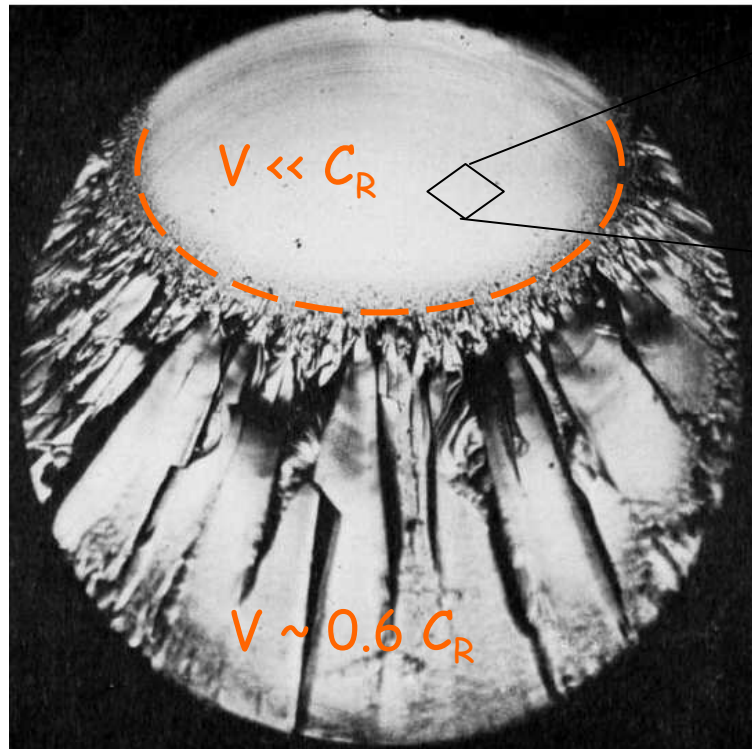


Trajectoire de fissures: prédictions

➤ Surfaces de rupture lisses

Surface de rupture par AFM
(Crédit, F. Celarié)

Faciès de rupture dans le verre:
Observations (Johnson Holloway, 1966)



Faciès rugueux !!!



PLAN DE L'EXPOSÉ

Mécanique de la rupture: Théorie **continue**

Rôle des défauts, failles de Griffith

Propagation de fissures: critère de stabilité,
équation de mouvement, trajectoire..

Rôle du désordre microstructural:
Description **stochastique**

Fissuration intermittente

ténacité effective

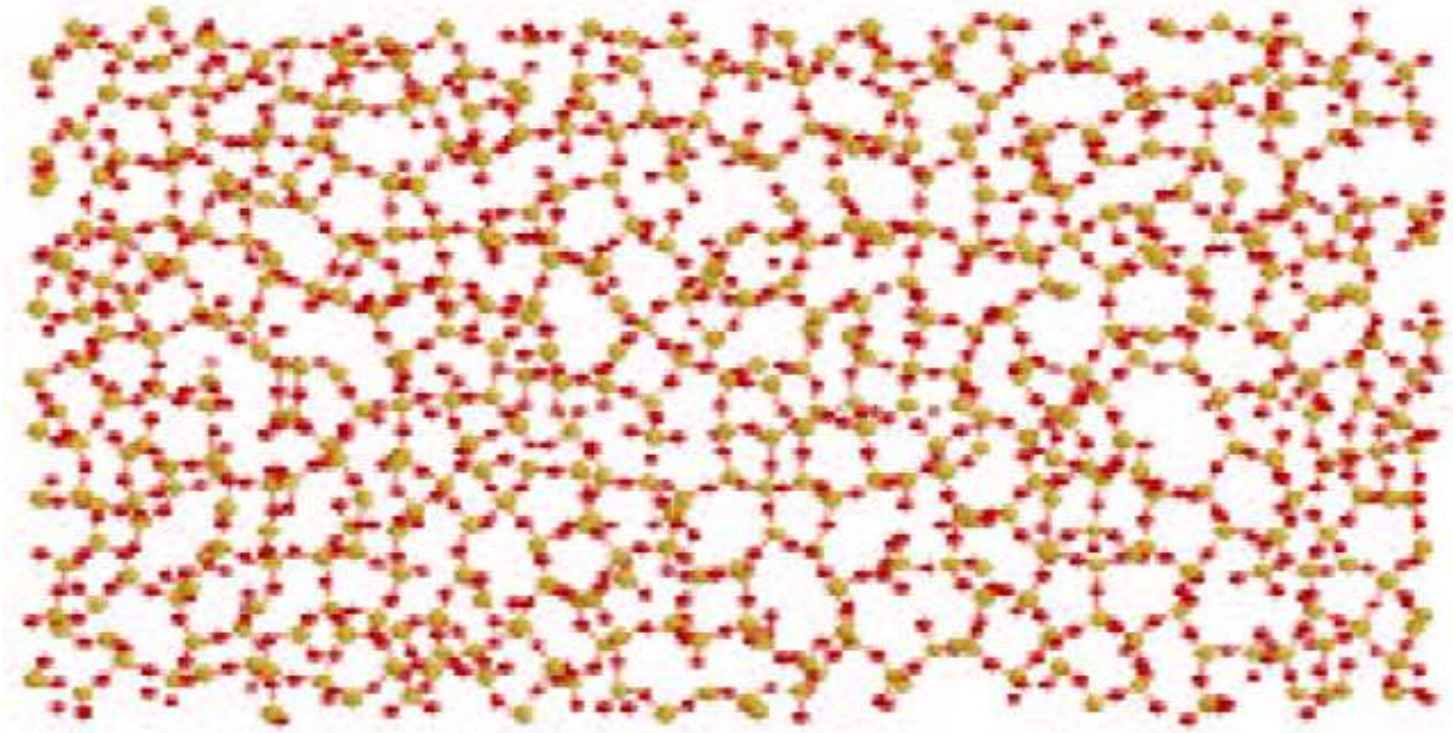
Faciès de rupture

Instabilité dynamique, branchement



Structure du verre

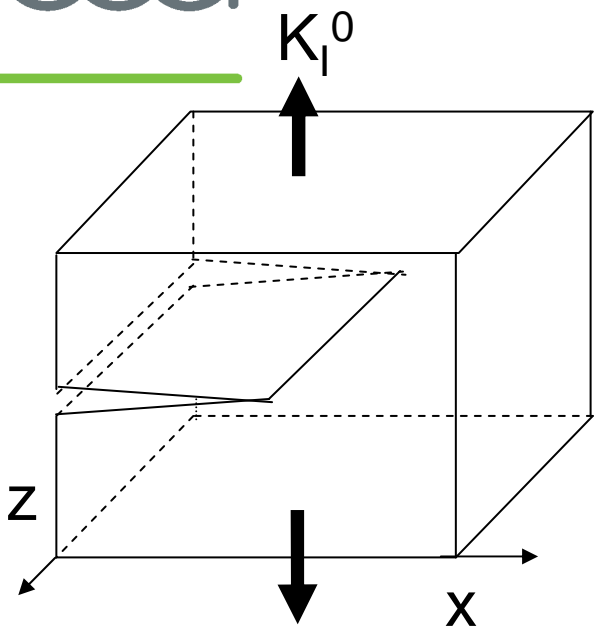
Coupe de 5 Å d'un verre de Silice simulé (crédit Van Brutzel)



- Hétérogène aux échelles sub-nanométriques
- Répercussions macroscopiques sur la rupture?

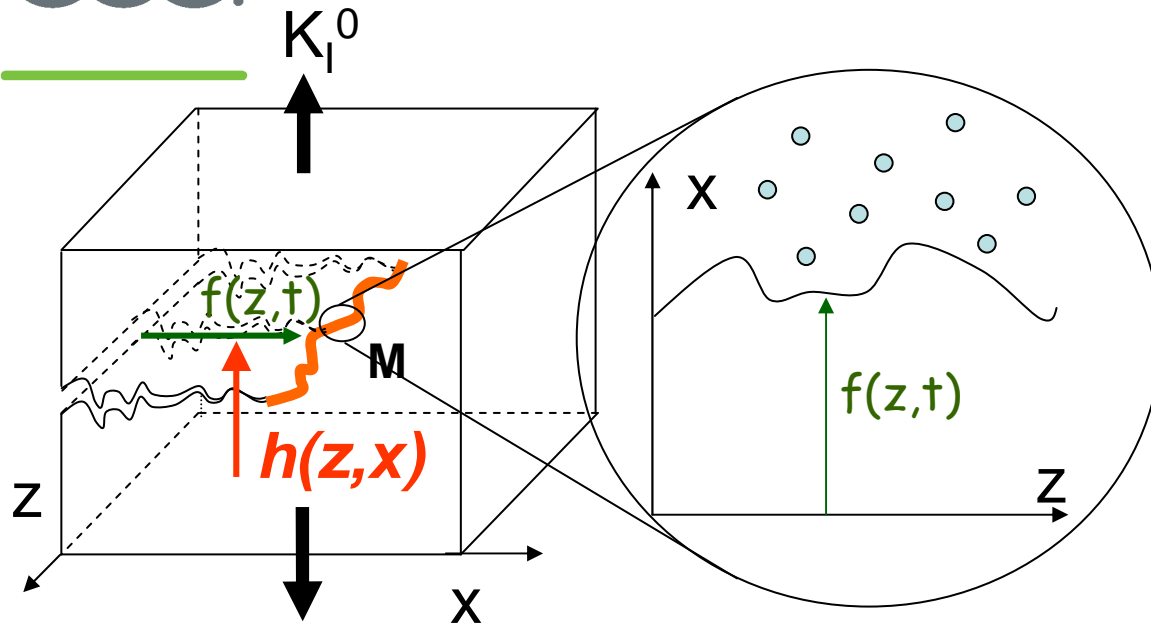


Désordre & equation de mouvement





Désordre & equation de mouvement



Faible désordre

$$\mu \frac{\partial f(z,t)}{\partial t} = K_I(M) - K_{Ic}(M)$$

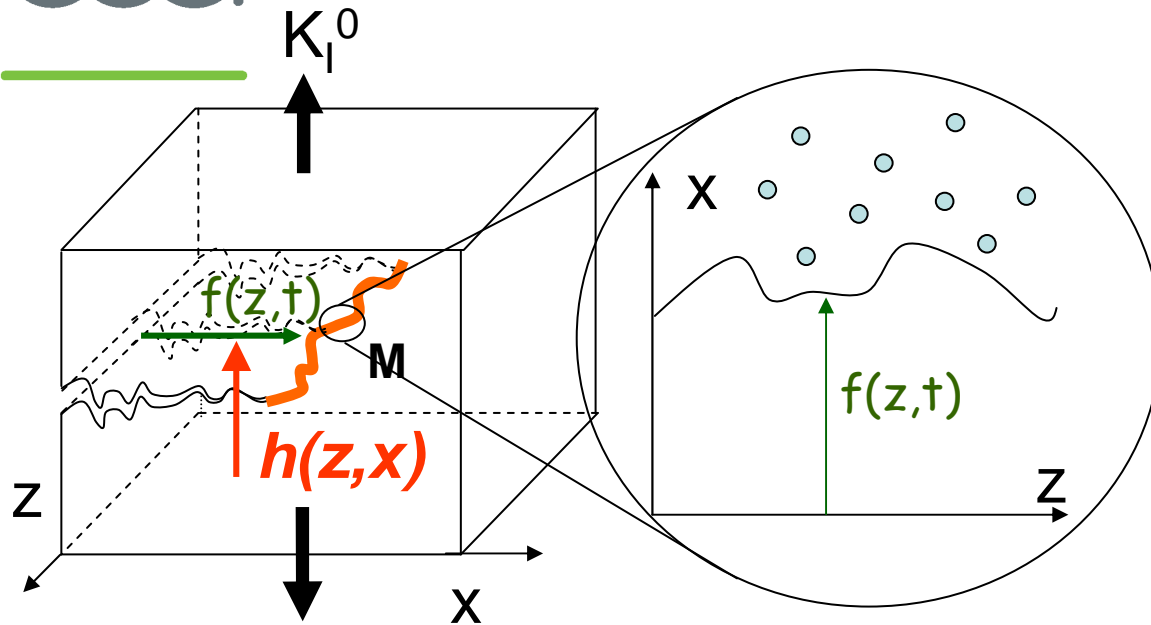
$$K_{Ic}(M) = K_{Ic}^0 (1 + \eta(z, f(z,t)))$$

$$K_I(M) = K_I^0 \left(1 + \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2} dz' \right) + \text{negligible}$$

Gao & Rice (89)



Désordre & equation de mouvement



Faible désordre

$$\mu \frac{\partial f(z,t)}{\partial t} = (K_I^0 - K_{Ic}^0) + \frac{1}{2\pi} K_I^0 \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z' - z)^2} dz' + K_{Ic}^0 \eta(z, f(z,t))$$

Equation de mouvement d'une ligne "élastique longue portée" en propagation dans un matériau aléatoire

Schmittbuhl et al. (95), Ramanathan et al. (97)



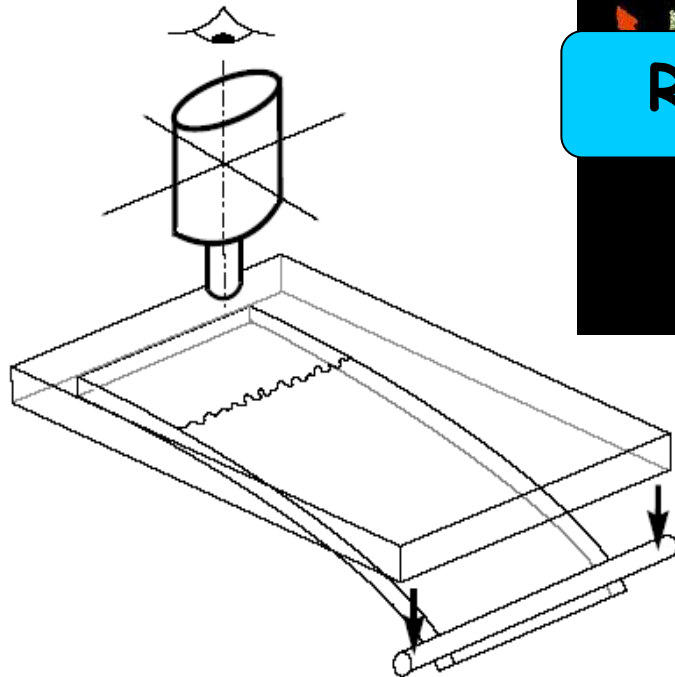
Fracture mechanics for ideal brittle materials...

Prediction: regular continuous crack growth velocity

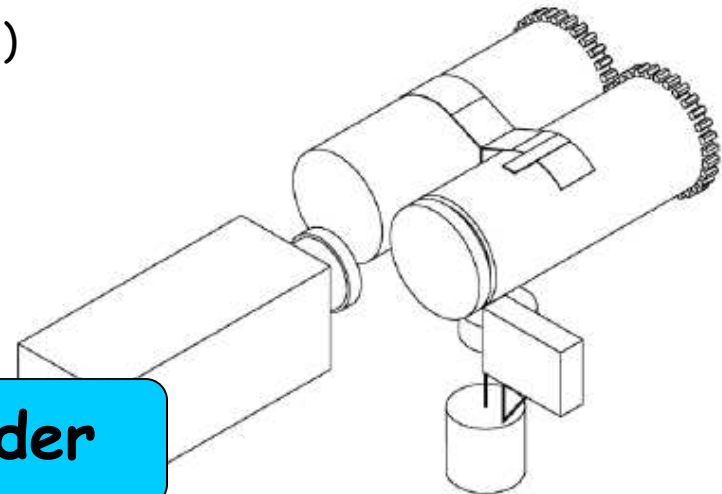
earthquakes
(from Sethna et al, 2001)



Interfacial crack



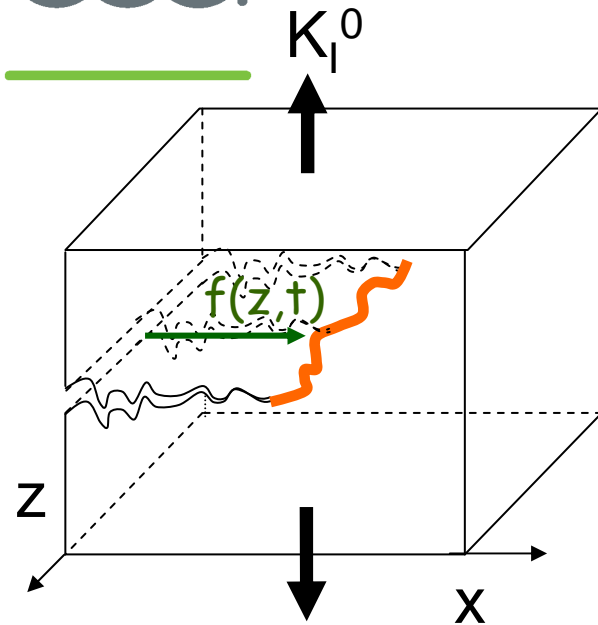
Role of disorder



Paper peeling
(Kovoisto et al. PRL, 2007)

Observation: intermittent crack growth with sudden random jump of all sizes !!!

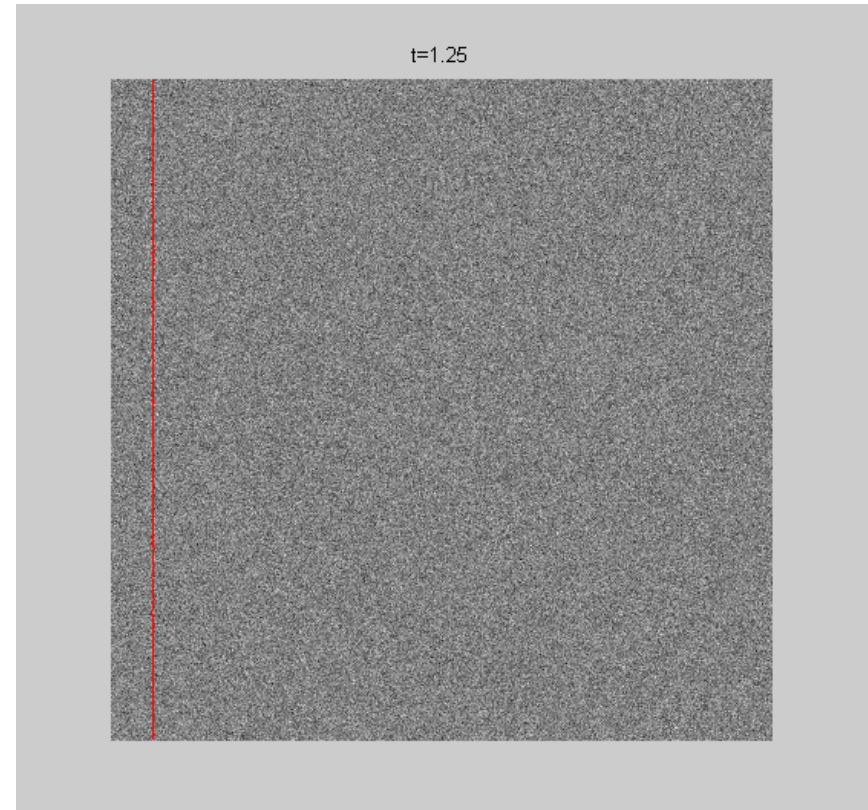
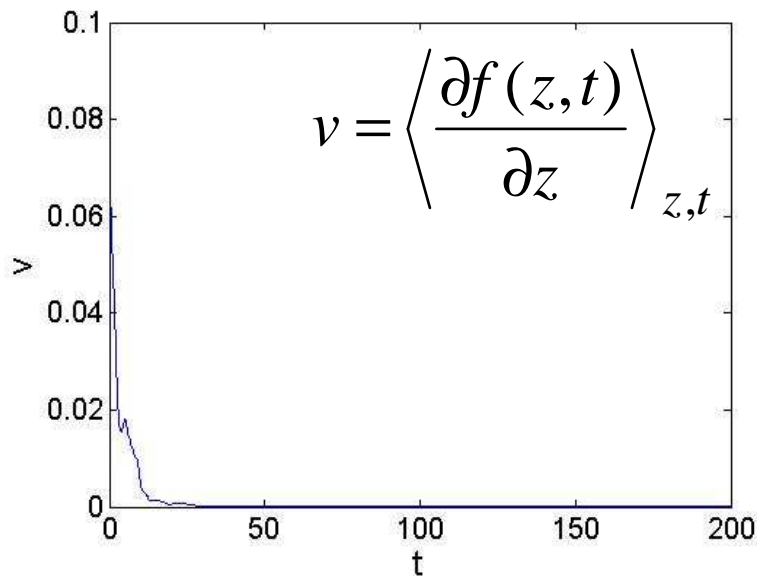
Désordre & equation de mouvement



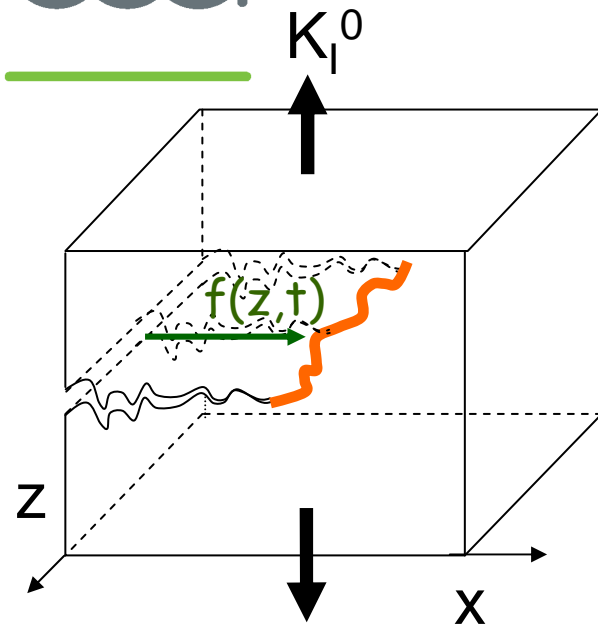
$$\mu \frac{\partial f(z,t)}{\partial t} = (K_I^0 - K_{Ic}^0) + \frac{1}{2\pi} K_I^0 \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2} dz' + K_{Ic}^0 \eta(z, f(z,t))$$

$$F = K_I^0 - K_{Ic}^0$$

$$F < F_c$$



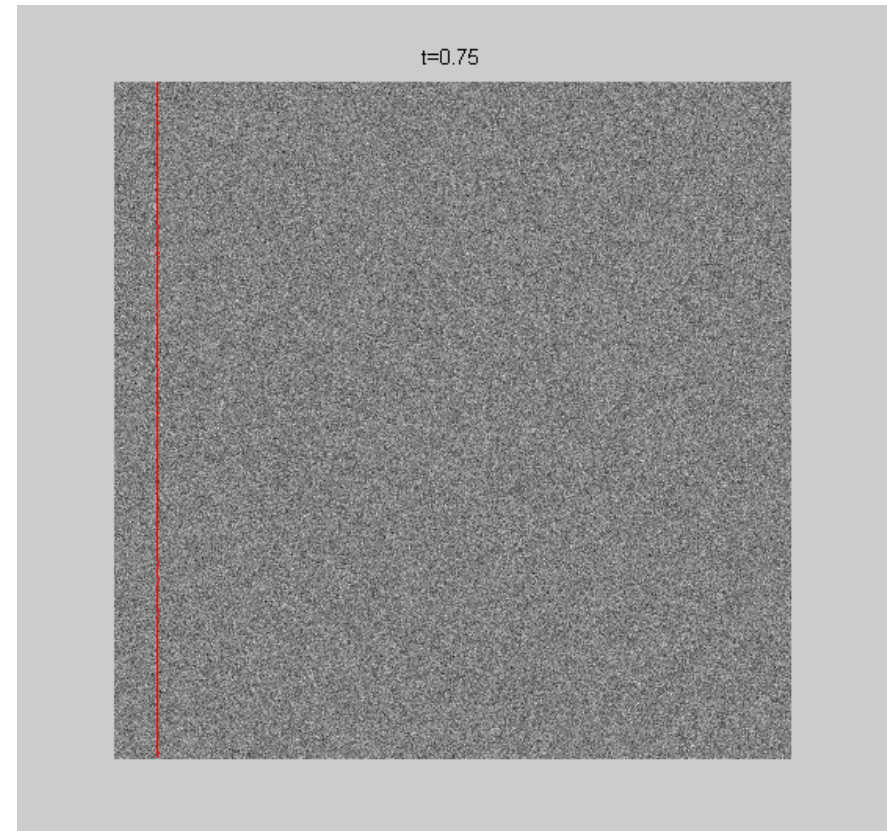
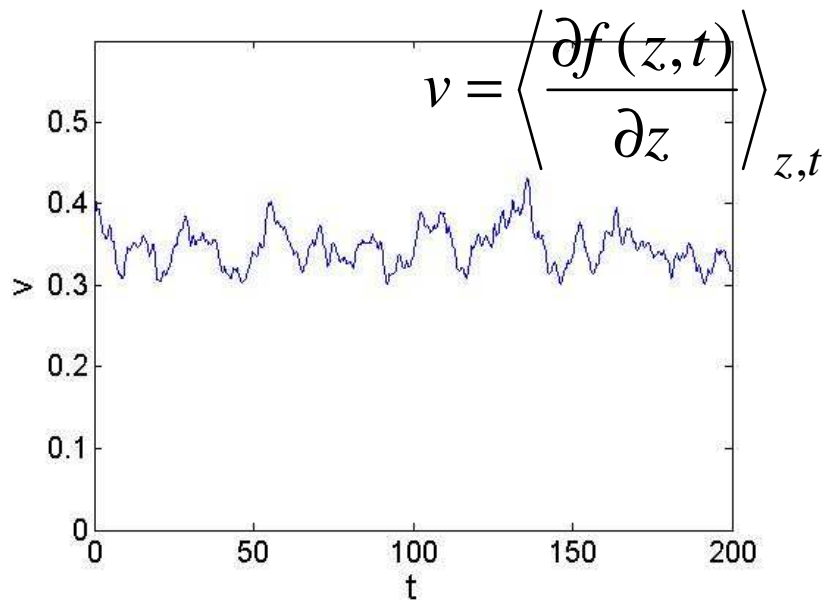
Désordre & equation de mouvement



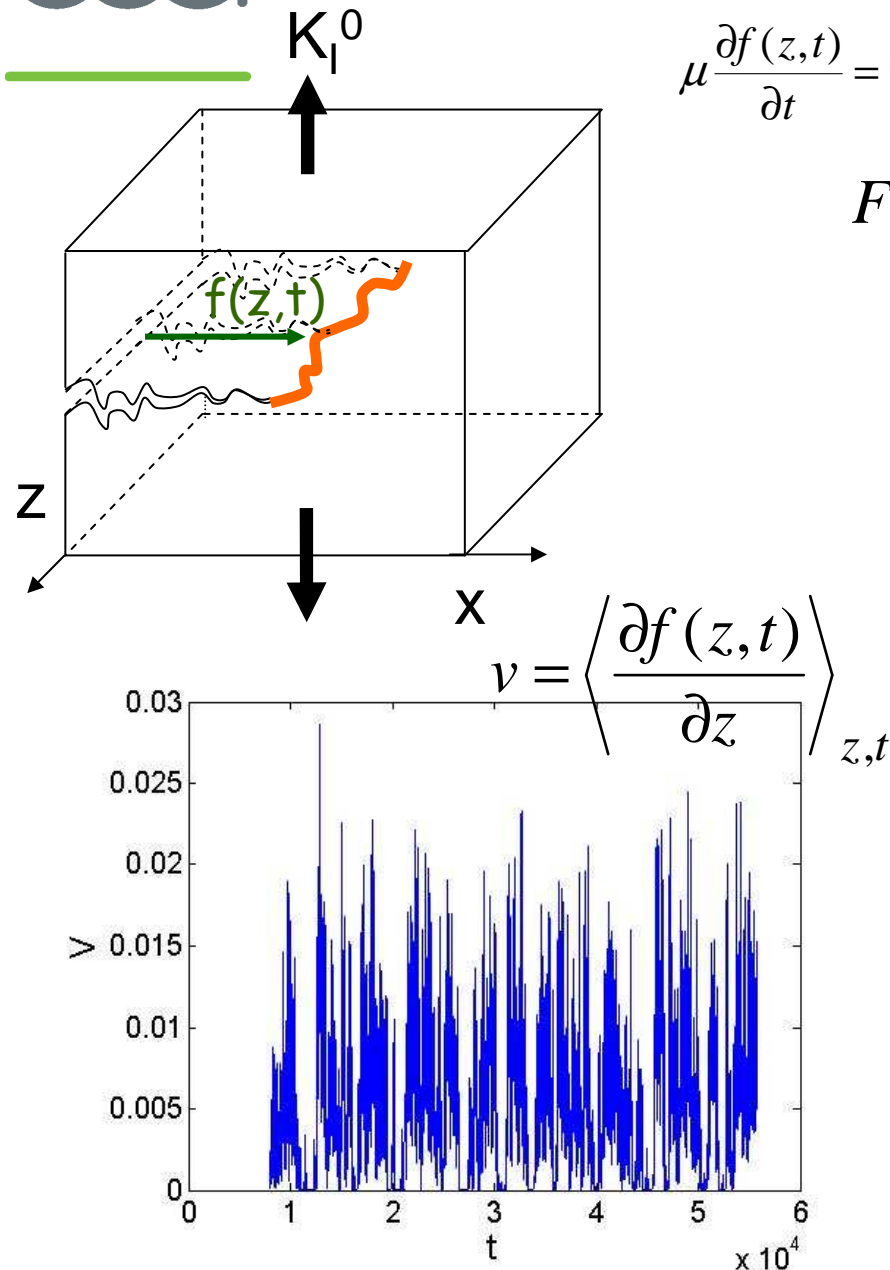
$$\mu \frac{\partial f(z,t)}{\partial t} = (K_I^0 - K_{Ic}^0) + \frac{1}{2\pi} K_I^0 \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2} dz' + K_{Ic}^0 \eta(z, f(z,t))$$

$$F = K_I^0 - K_{Ic}^0$$

$$F > F_c$$



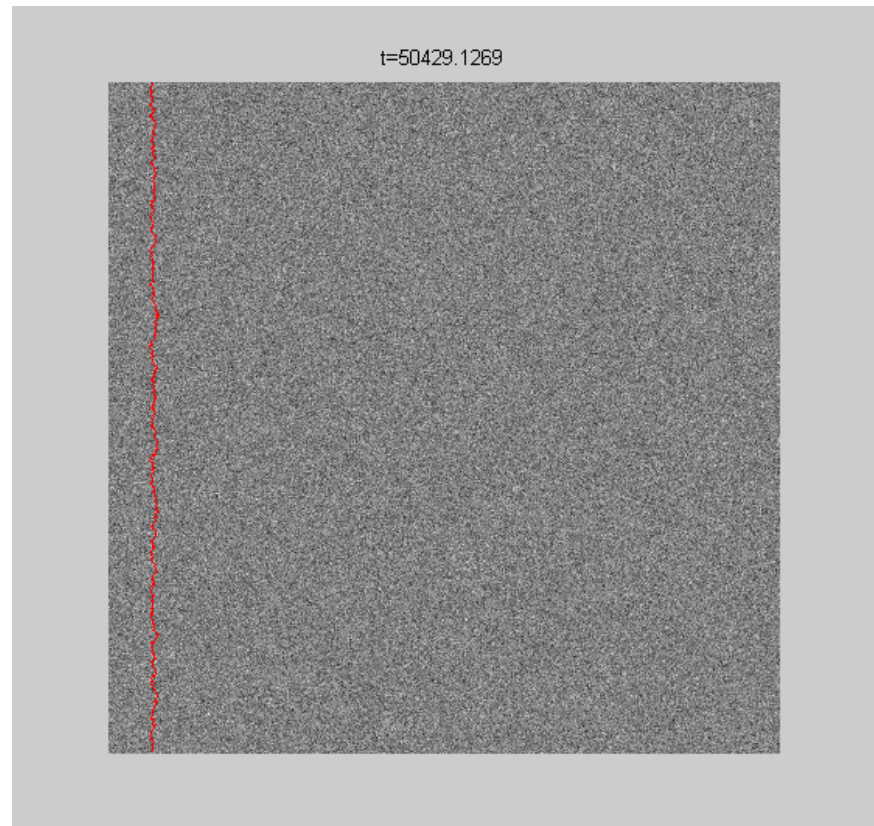
Désordre & equation de mouvement



$$\mu \frac{\partial f(z,t)}{\partial t} = (K_I^0 - K_{Ic}^0) + \frac{1}{2\pi} K_I^0 \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2} dz' + K_{Ic}^0 \eta(z, f(z,t))$$

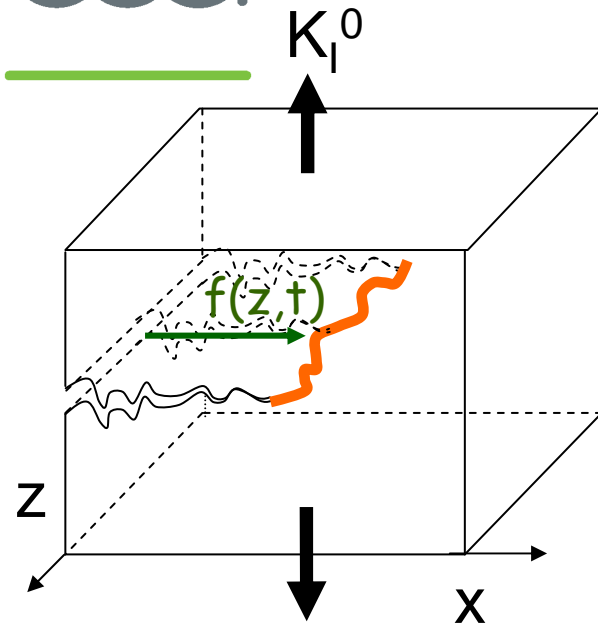
$$F = K_I^0 - K_{Ic}^0$$

$$F \sim F_c$$





Désordre & equation de mouvement



$$\mu \frac{\partial f(z,t)}{\partial t} = (K_I^0 - K_{Ic}^0) + \frac{1}{2\pi} K_I^0 \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2} dz' + K_{Ic}^0 \eta(z, f(z,t))$$

Transition critique

Ertas & Kardar (94)

Param. ordre

$$v = \left\langle \frac{\partial f(z,t)}{\partial z} \right\rangle_{z,t}$$

Param. control

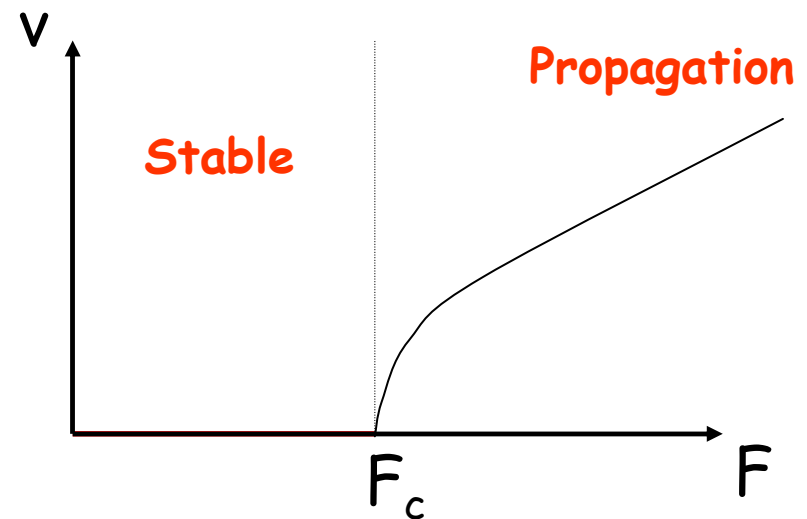
$$F = K_I^0 - K_{Ic}^0$$

$F < F_c$ $v = 0$ fissure **stable**

$F \gg F_c$ $v \propto F$ fissure **en propagation**

$F \gtrsim F_c$ Point **critique, universalité**

$$K_{Ic}^{eff} = K_{Ic}^0 + F_c$$





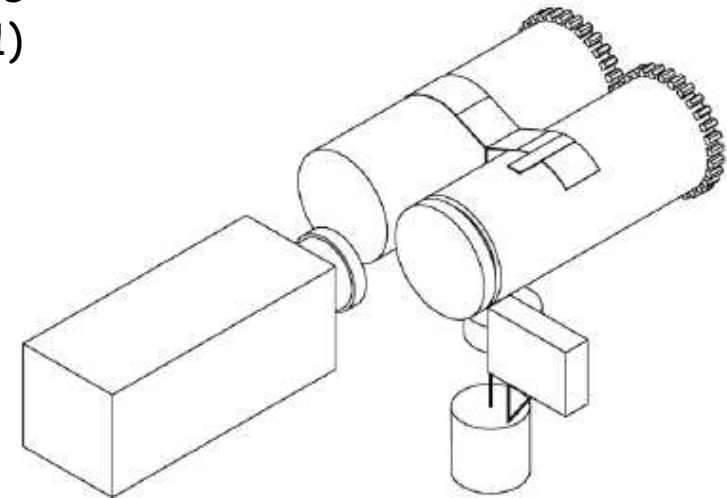
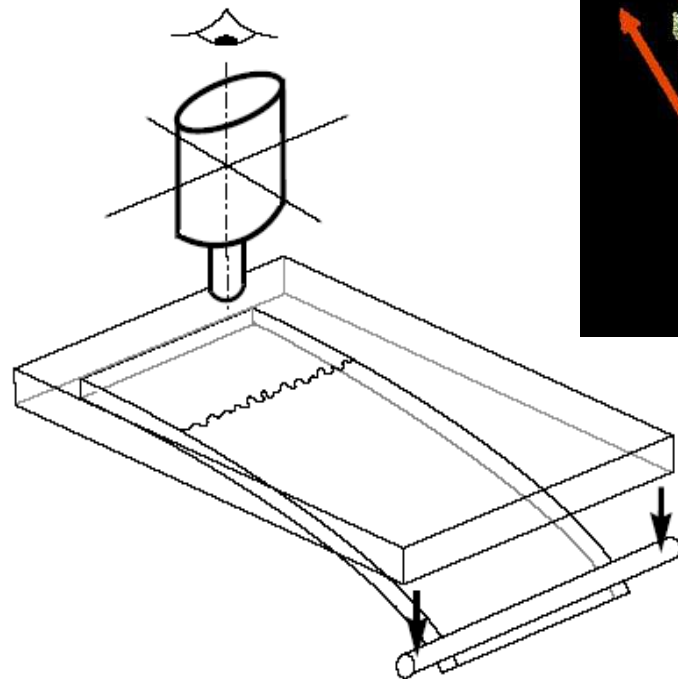
Désordre & equation de mouvement

Prediction: fissuration quasi-statique intermittente

Tremblement de terre
(from Sethna et al, 2001)



Fissure interfaciale dans le Plexiglas
(Maloy et al, 2006)



Epluchage du papier
(Kovoisto et al. PRL, 2007)

Observée dans de nombreux matériaux fragiles hétérogènes !!!



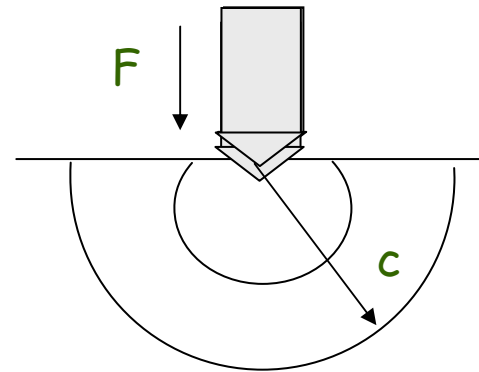
Désordre & equation de mouvement

Applications: Statistique des ténacités

Charles, Hild, Roux, Vandembroucq, 2003, 2004, 2006

Limite thermodynamique:

$$K_{Ic}^{eff} = cte = K_{Ic}^{\infty}$$



$$L \propto c$$

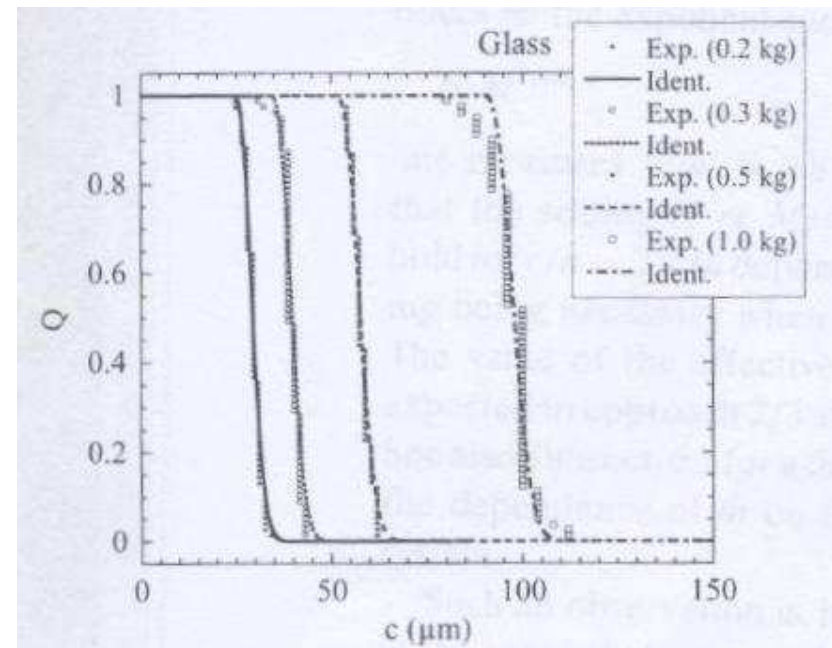
$$K_{Ic}^{eff} \propto \frac{F}{c^{3/2}}$$

L fini: distribution statistique

$$K_{Ic}^{eff} < K_{Ic}^{\infty}$$

$$\sigma = \left\langle \left(K_{Ic}^{eff} - \langle K_{Ic}^{eff} \rangle \right)^2 \right\rangle^{1/2} \propto L^{-1/\nu}$$

$$P(K_{Ic}^{eff}, L) = L^{1/\nu} \psi \left(\frac{K_{Ic}^{\infty} - K_{Ic}^{eff}}{\sigma} \right)$$



Longueur d'arrêt dans les tests d'indentation



PLAN DE L'EXPOSÉ

Mécanique de la rupture: Théorie **continue**

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ténacité effective

Faciès de rupture

Instabilité dynamique, branchement

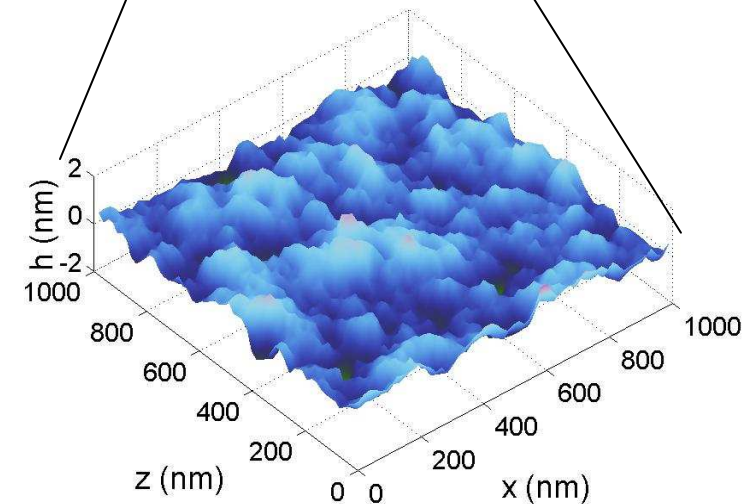
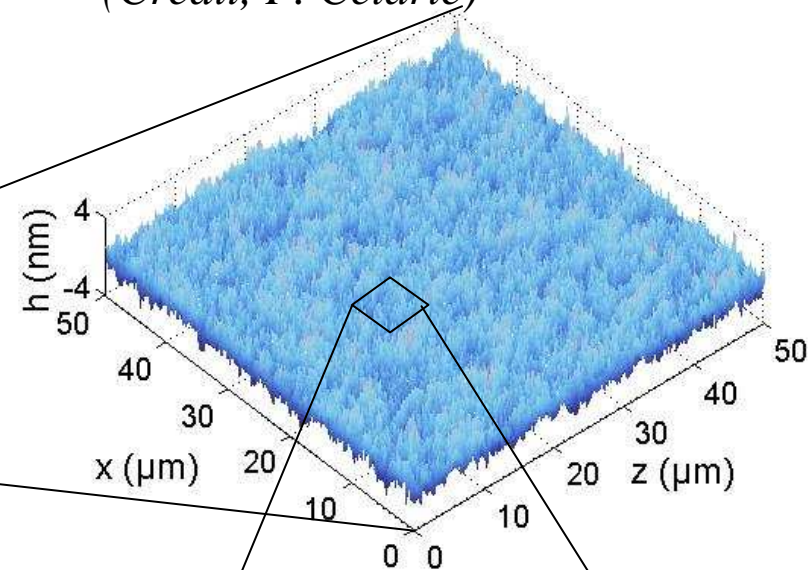
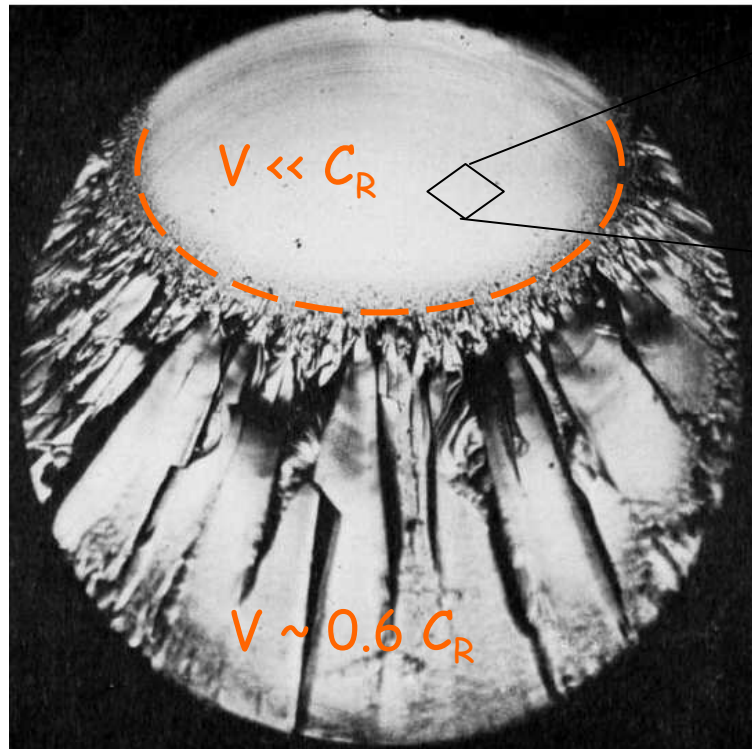


Trajectoire de fissures: prédictions

➤ Surfaces de rupture lisses

Surface de rupture par AFM
(Crédit, F. Celarié)

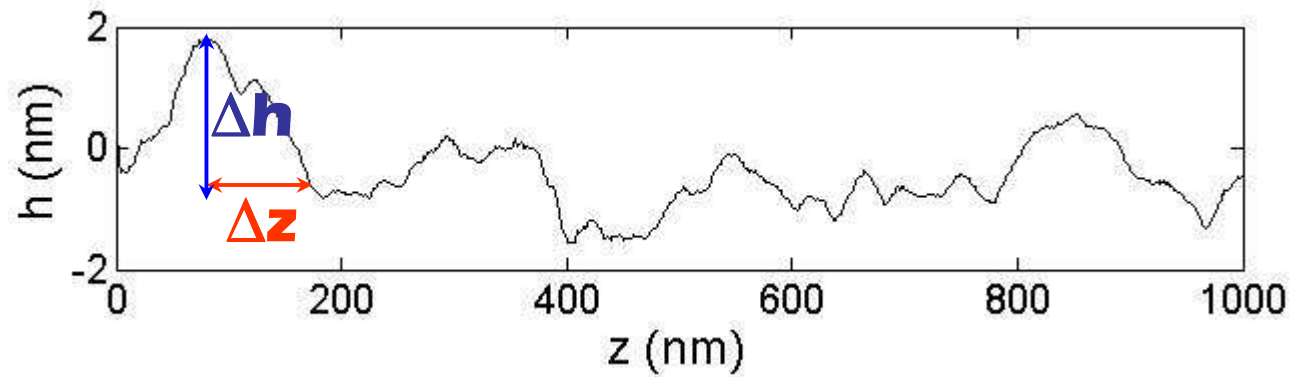
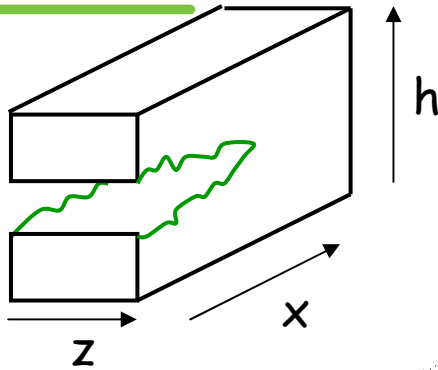
Faciès de rupture dans le verre:
Observations (Johnson Holloway, 1966)



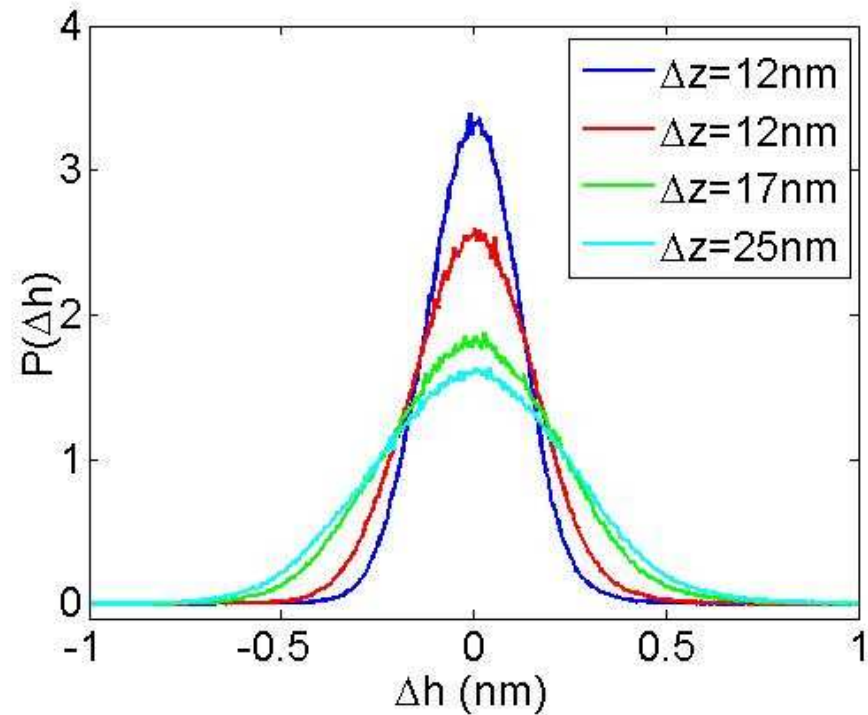
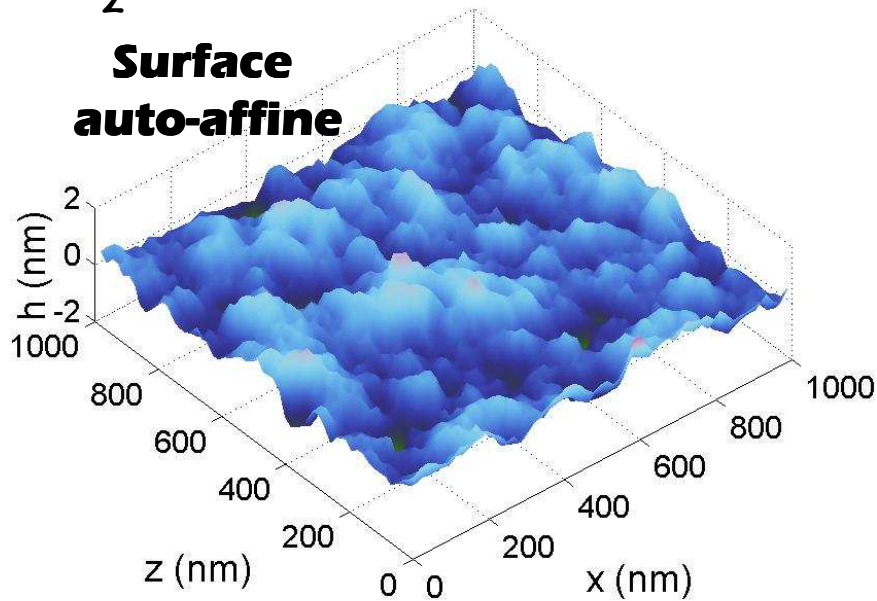
Faciès rugueux !!!

Faciès de rupture: lois d'échelle

Mandelbrot et al Nature 84; Bouchaud et al. EPL 90; Maloy et al. PRL 92,...



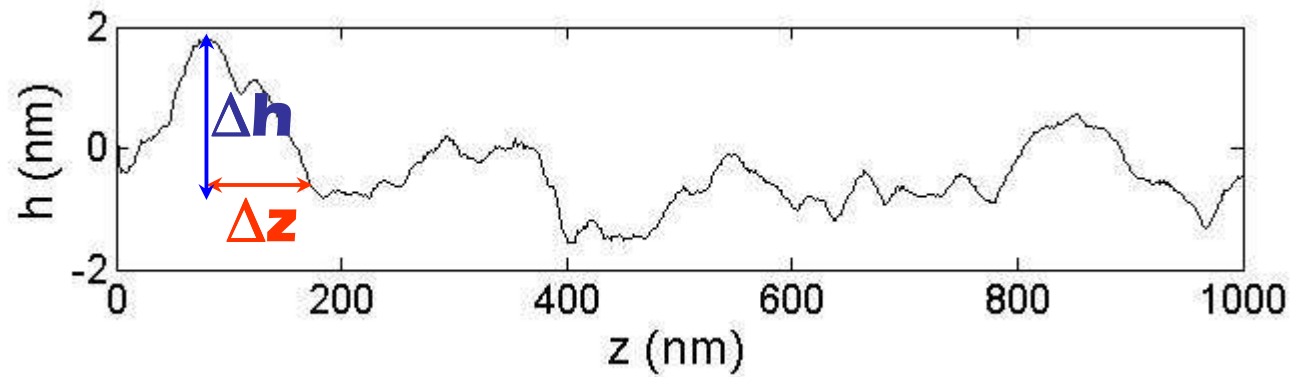
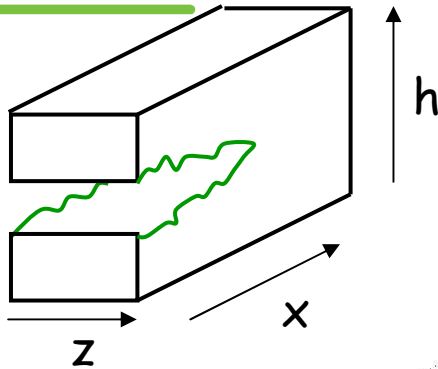
Surface auto-affine



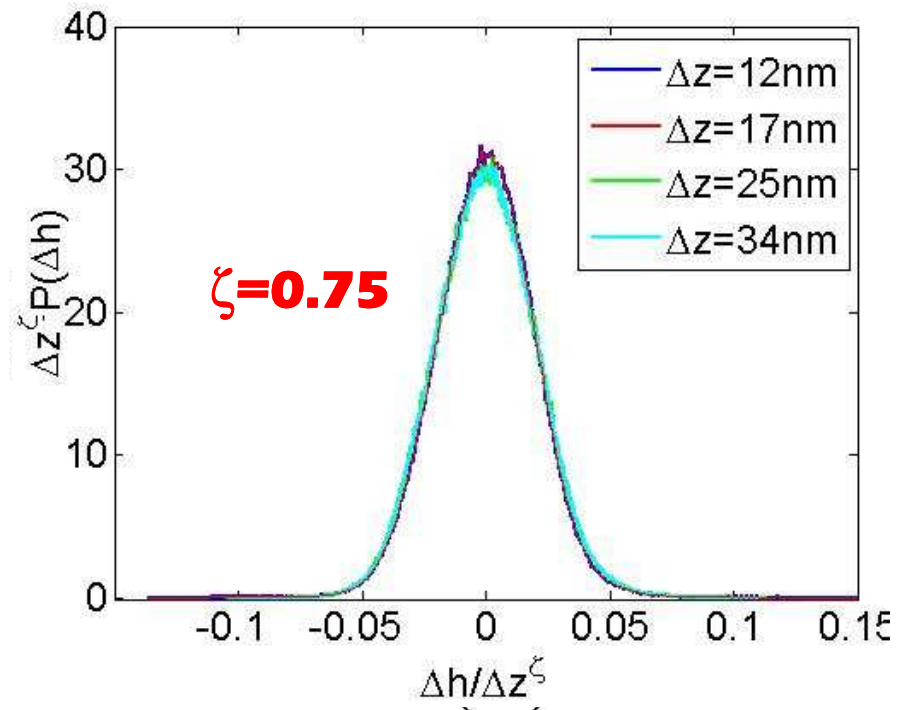
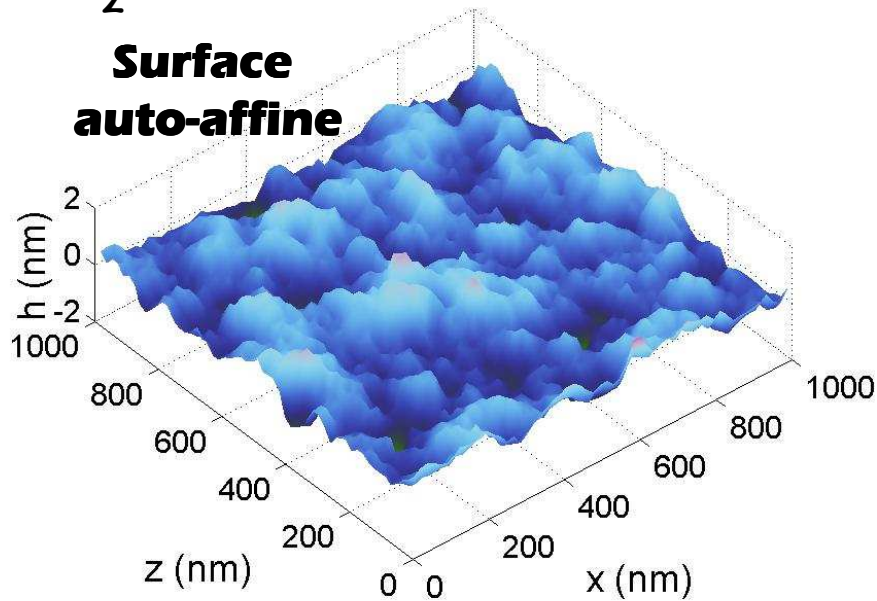


Faciès de rupture: lois d'échelle

Mandelbrot et al Nature 84; Bouchaud et al. EPL 90; Maloy et al. PRL 92,...



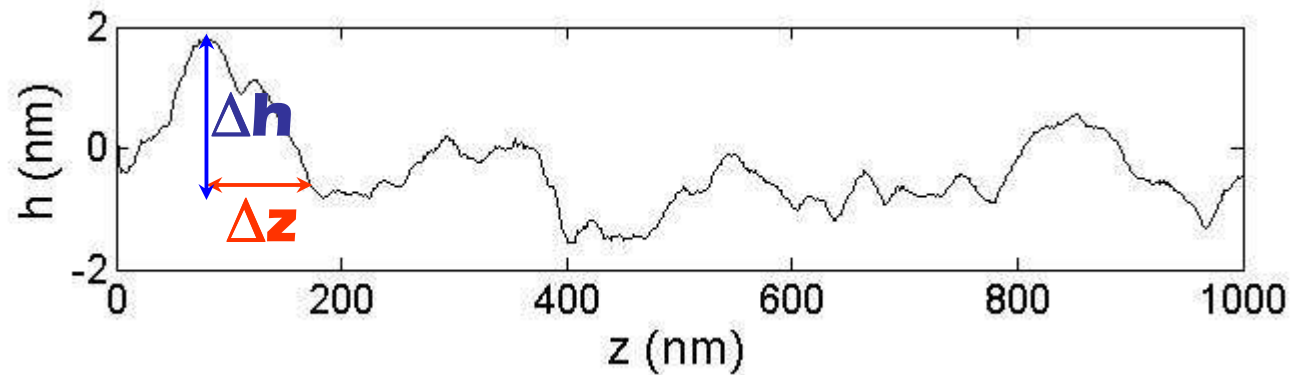
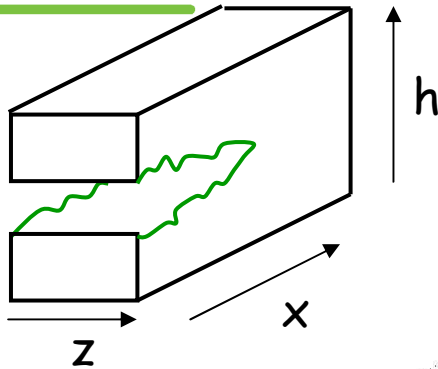
Surface auto-affine



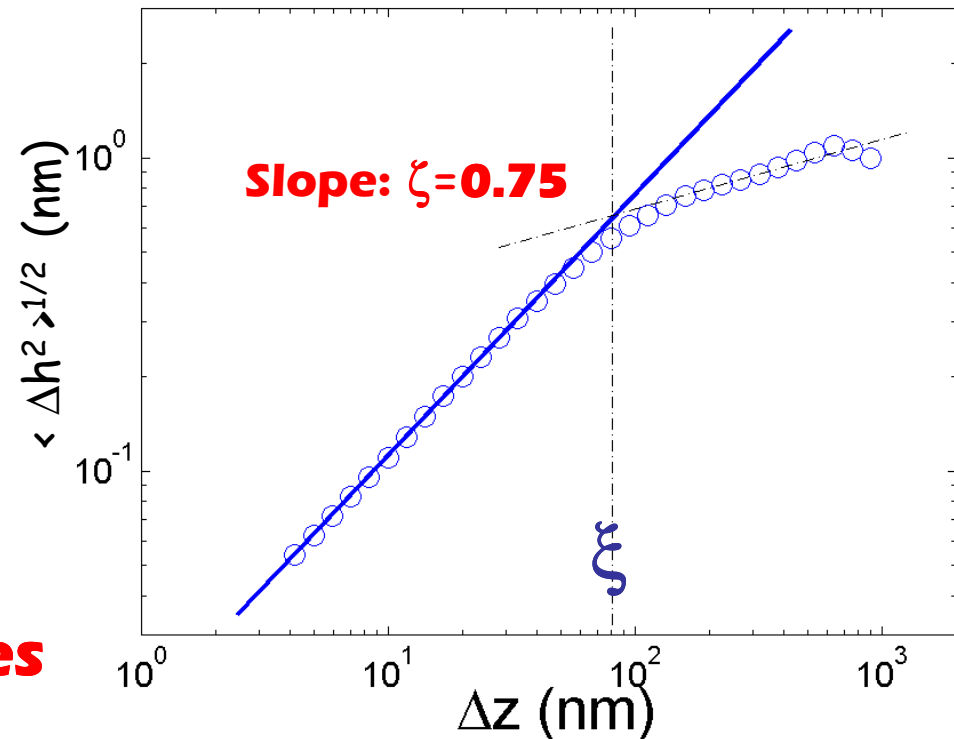
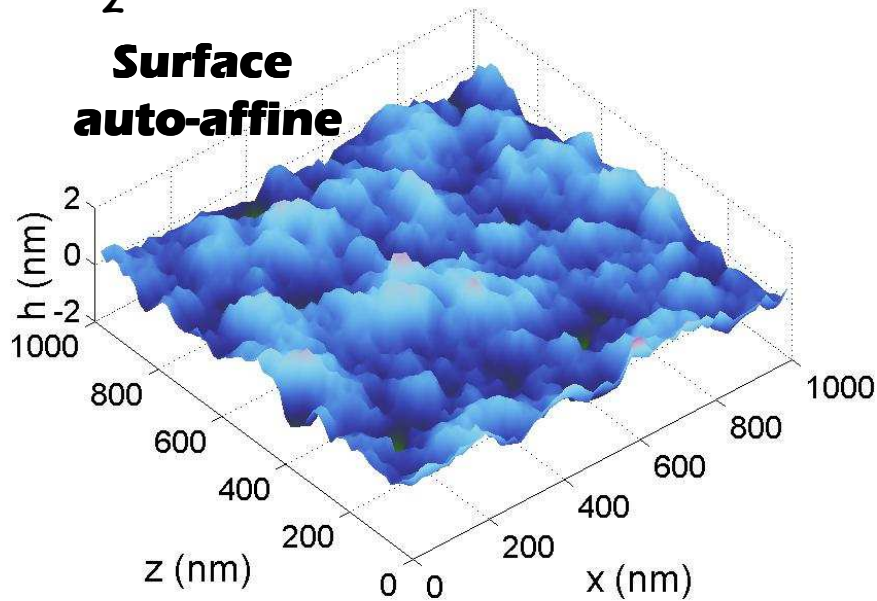


Faciès de rupture: lois d'échelle

Mandelbrot et al Nature 84; Bouchaud et al. EPL 90; Maloy et al. PRL 92,...



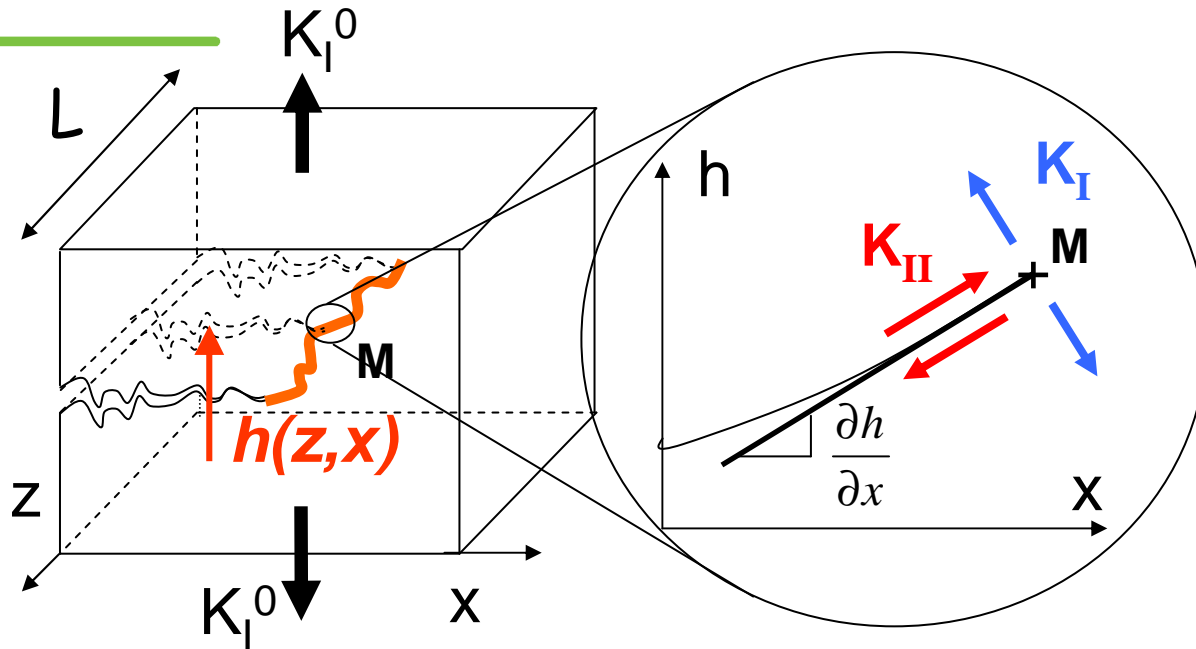
Surface auto-affine



$\zeta \sim 0.75$ universel dans de nombreux matériaux fragiles



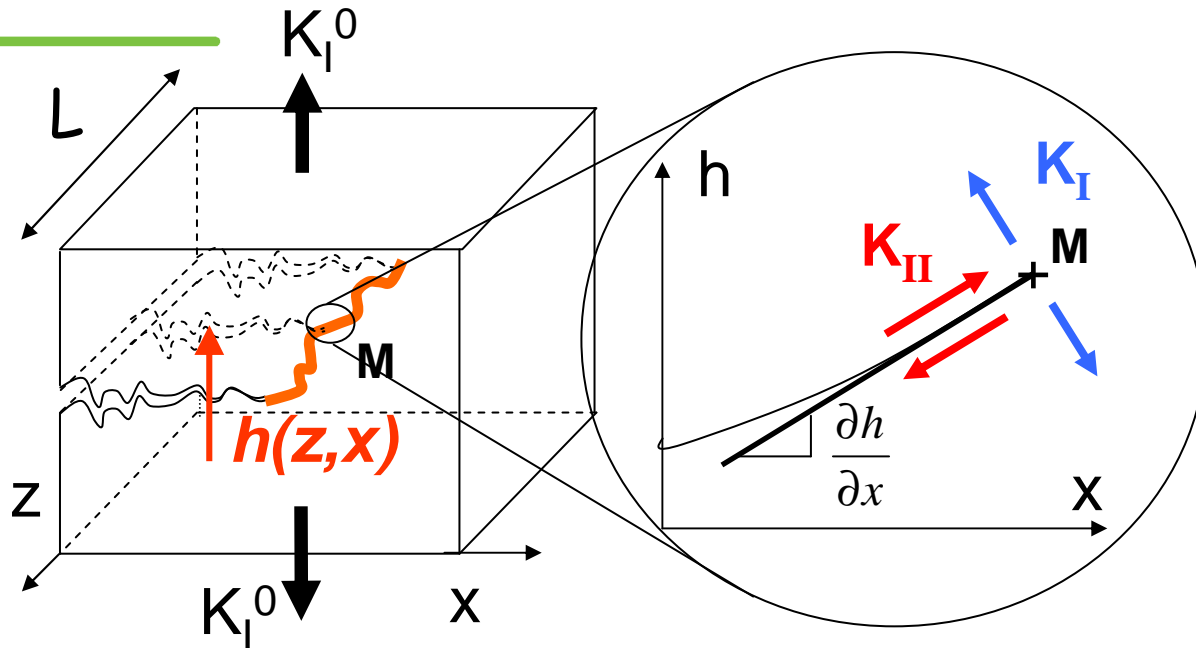
Désordre & equation de trajectoire



Local symmetry Principle

$$K_{II}(M) = 0$$

B. Cotterell et J. Rice (1980)



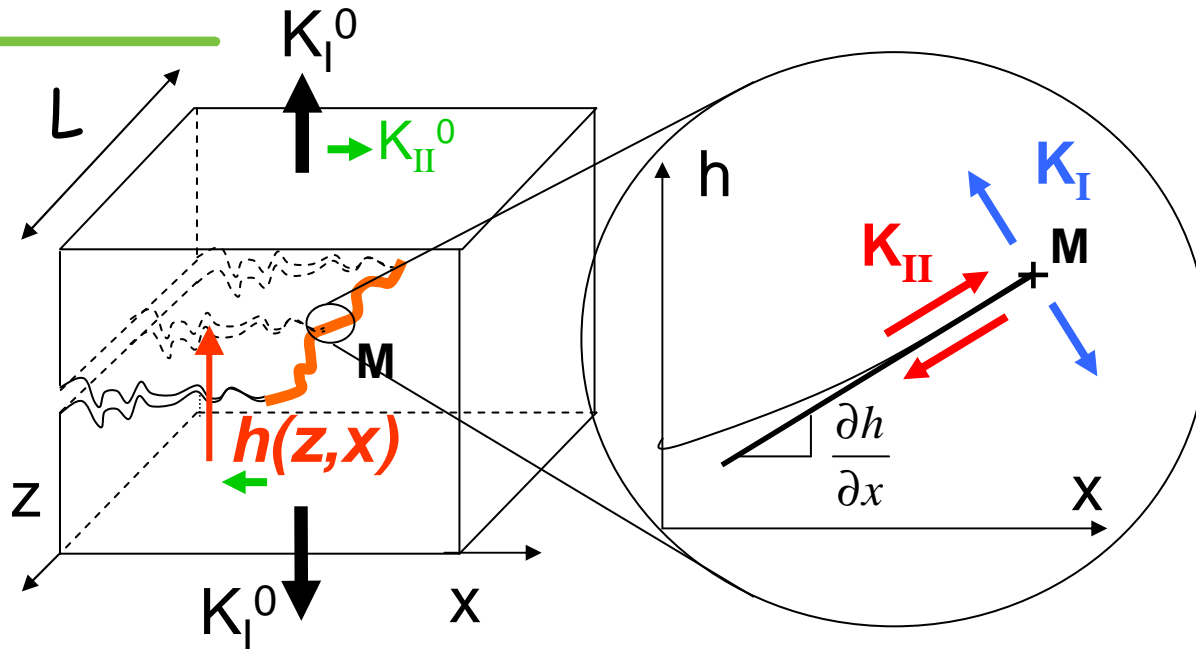
Loading perturbation induced by out-of-plane roughness

$$K_{II}(M) = 0 = 2K_I^0 \frac{\partial h(x, z)}{\partial x} - 2K_I^0 A \int_{-\infty}^{+\infty} \frac{h(x, z') - h(x, z)}{(z' - z)^2} dz' + \text{negligible}$$

Larralde & Ball (1995), Movchan et al. (1998)



Désordre & equation de trajectoire

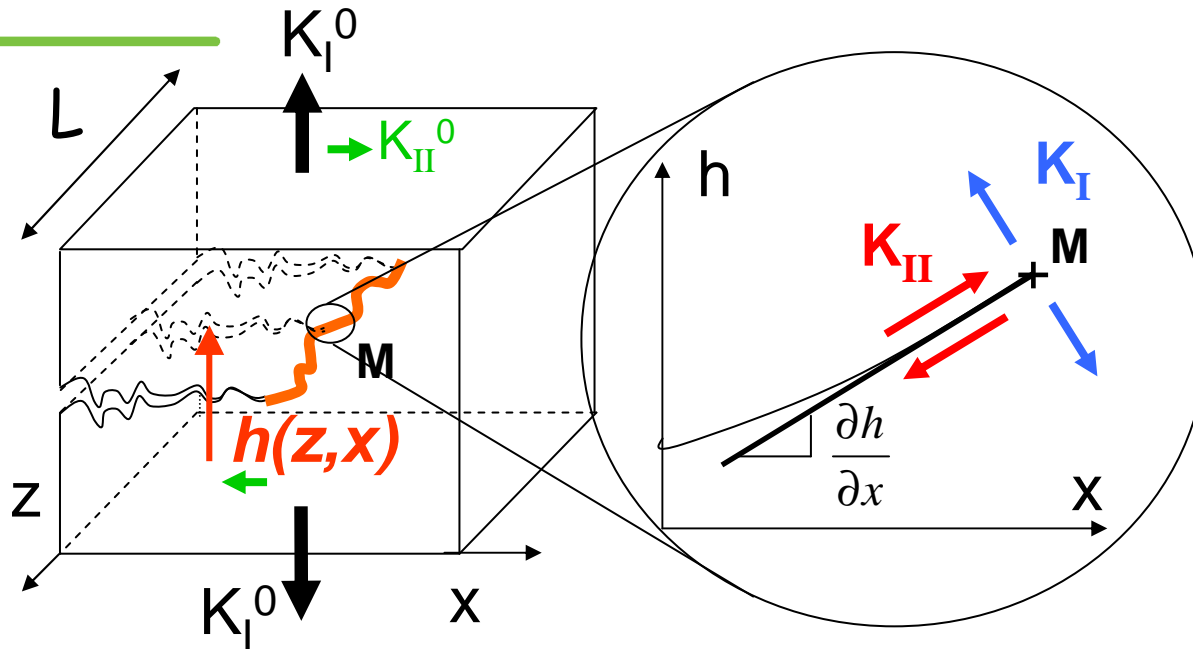


$$K_{II}(M) = 0 = 2K_I^0 \frac{\partial h(x,z)}{\partial x} - 2K_I^0 A \int_{-\infty}^{+\infty} \frac{h(x,z') - h(x,z)}{(z'-z)^2} dz' + \text{negligible}$$

+ K_{II}^0
Slight experimental misalignment



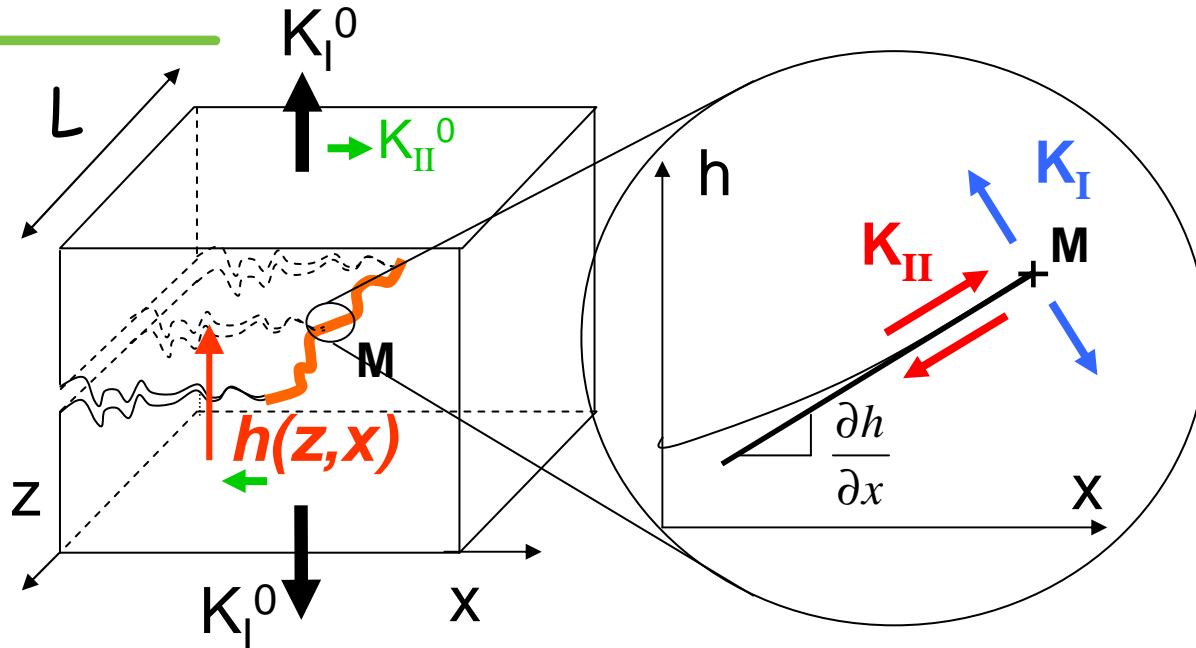
Désordre & equation de trajectoire



$$\begin{aligned}
 K_{II}(M) = 0 &= 2K_I^0 \frac{\partial h(x, z)}{\partial x} - 2K_I^0 A \int_{-\infty}^{+\infty} \frac{h(x, z') - h(x, z)}{(z' - z)^2} dz' + \text{negligible} \\
 &+ K_{II}^0 + \eta(x, z, h(x, z)) \\
 &\quad \text{Material disorder}
 \end{aligned}$$

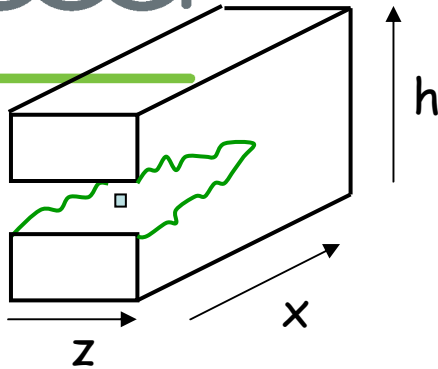


Désordre & equation de trajectoire



$$\frac{\partial h(z, x)}{\partial x} = -A \int_{-\infty}^{+\infty} \frac{h(z', x) - h(z, x)}{(z' - z)^2} dz' + \frac{K_{II}^0}{K_I^0} + \eta(z, x, h(z, x))$$

Désordre & équation de trajectoire



$$\frac{\partial h(z, x)}{\partial x} = -A \int_{-\infty}^{+\infty} \frac{h(z', x) - h(z, x)}{(z' - z)^2} dz' + \frac{K_{II}^0}{K_I^0} + \eta(z, x, y = h(z, x))$$

$$\langle \eta(z, x, y) \eta(z', x', y') \rangle = Df(|\vec{r} - \vec{r}'| / a)$$

fonction de structure: $G(\Delta z, \Delta x) = \langle (h(z + \Delta z, x + \Delta x) - h(z, x))^2 \rangle$

$$\delta h \ll a$$

$$\eta(x, z, h(x, z)) \sim \eta_q(z, h(x, z)) + \eta_+(z, x)$$

Profile selon z auto-affine:

*Kolton et al. (05), Tanguy et al. (98),
Rosso & Krauth (02), Duemmer & Krauth (07),*

$$G(\Delta z, \Delta x = 0) \propto \Delta z^{2\zeta} \quad \zeta = 0.4$$

$$\delta h \gg a$$

$$\eta(x, z, h(x, z)) \sim \eta_+(z, x)$$

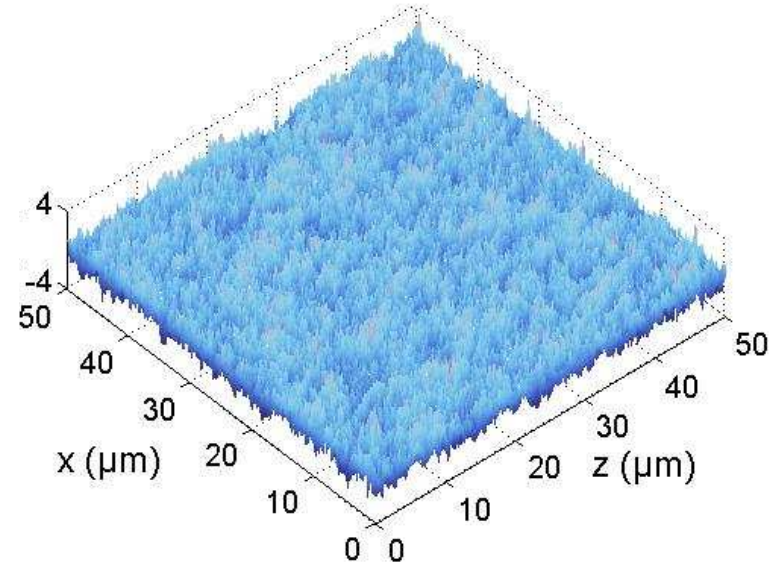
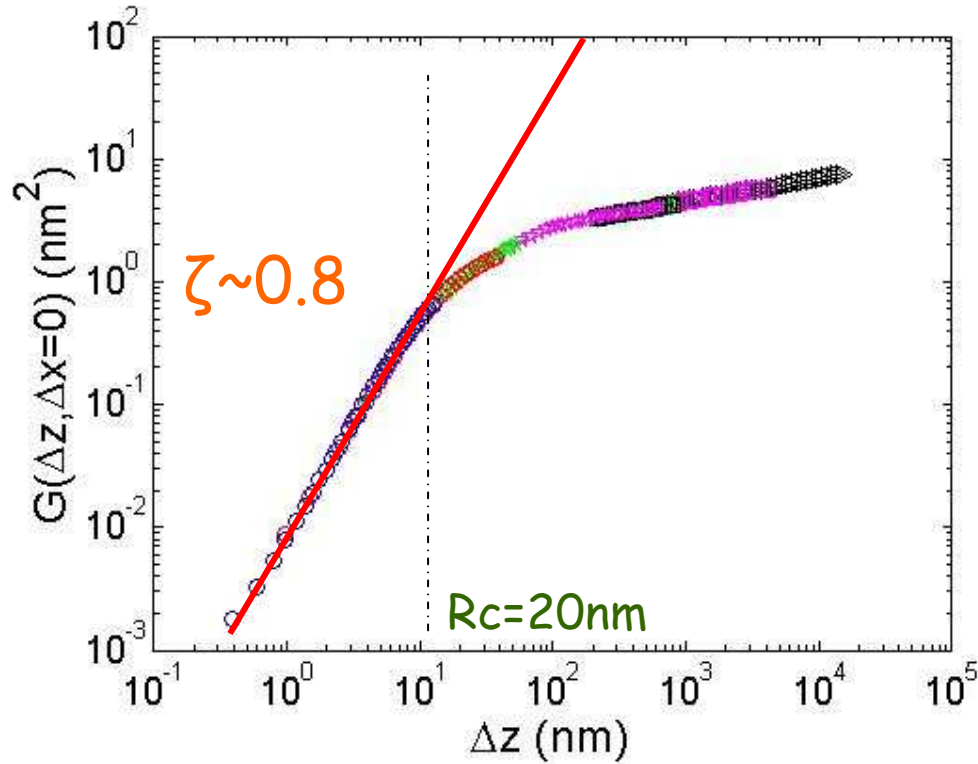
Rugosité logarithmique

Ramanathan, Ertas & Fisher (97)

$$G(\Delta z, 0) \sim 0.318 \frac{D}{A} a^2 \cdot \log\left(\frac{\Delta z}{a}\right) + 0.549 \frac{D}{A} a^2$$



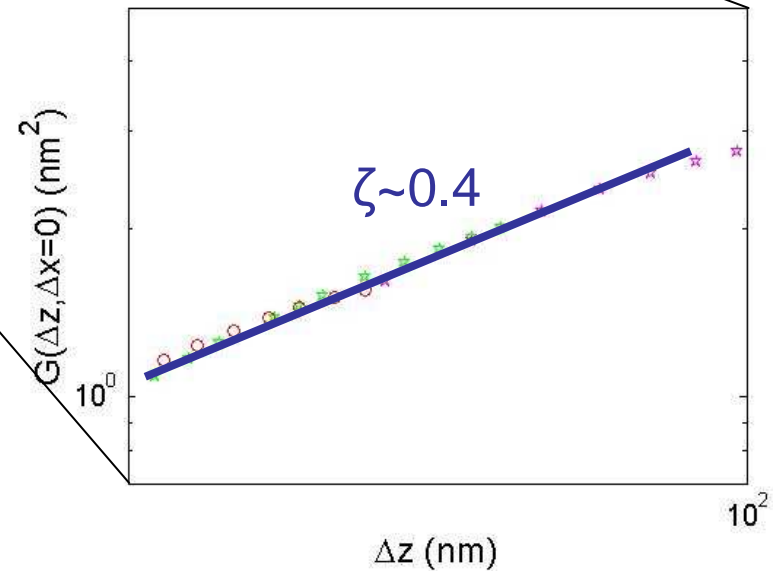
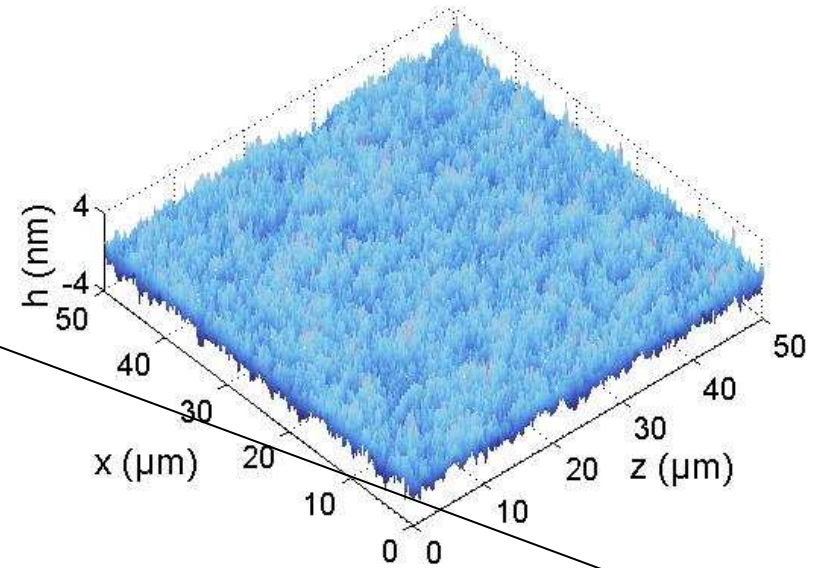
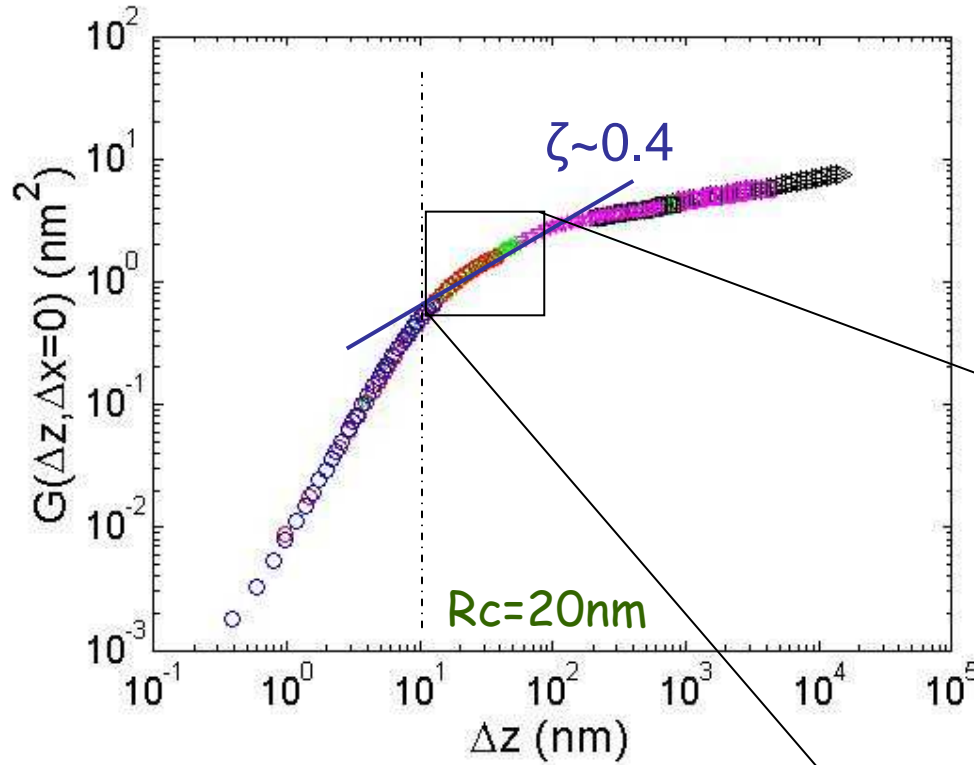
Désordre & équation de trajectoire



Contradiction apparente

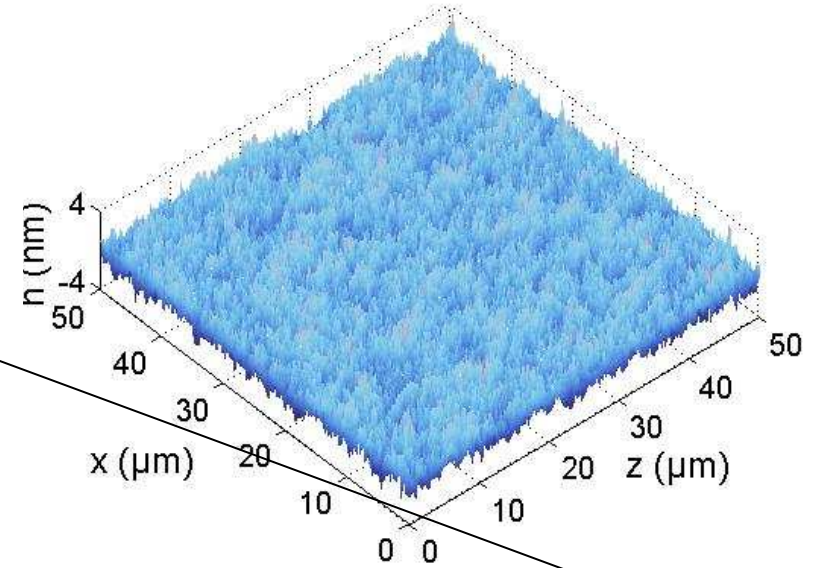
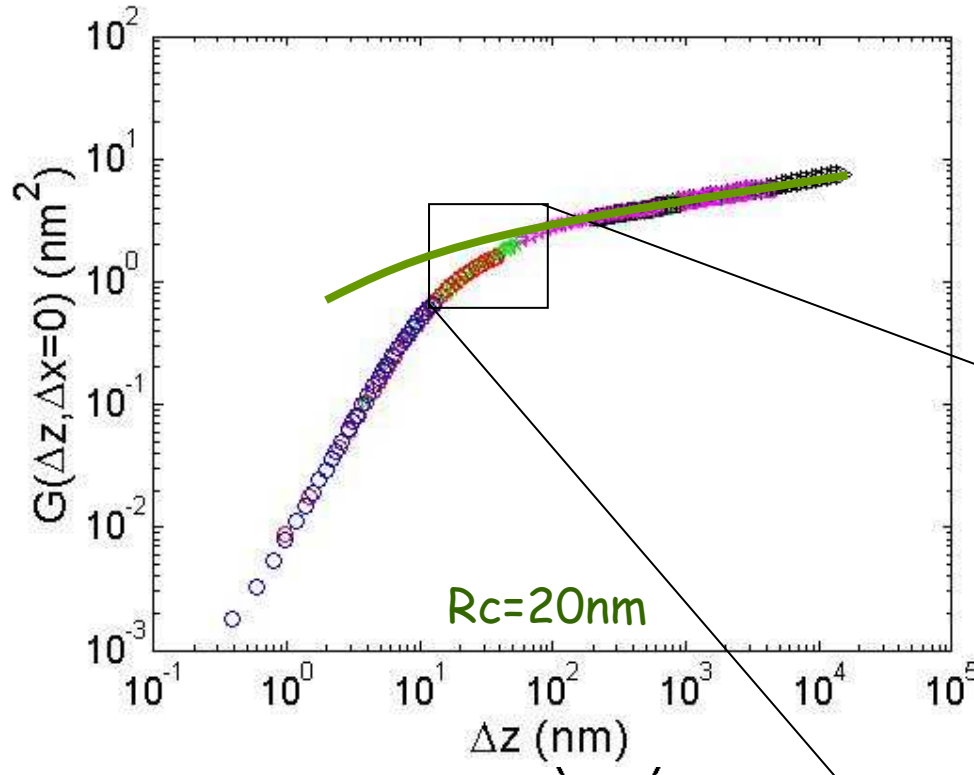


Désordre & équation de trajectoire



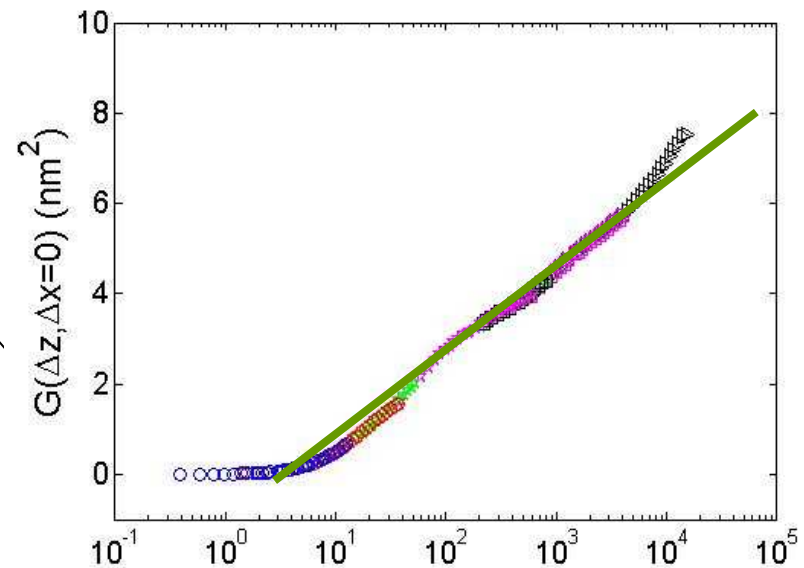


Désordre & équation de trajectoire



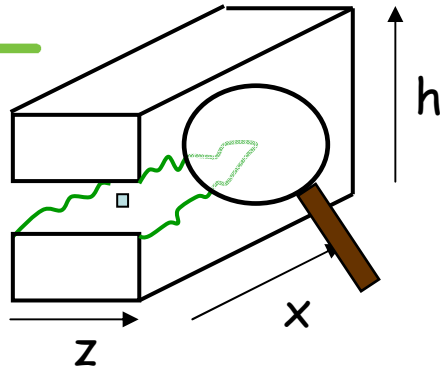
$$G(\Delta z, 0) \sim 0.318 \frac{D}{A} a^2 \cdot \log\left(\frac{\Delta z}{a}\right) + 0.549 \frac{D}{A} a^2$$

Avec $a=Rc=20\text{nm}$, $D/A=5 \times 10^{-3} \text{ nm}^{-2}$



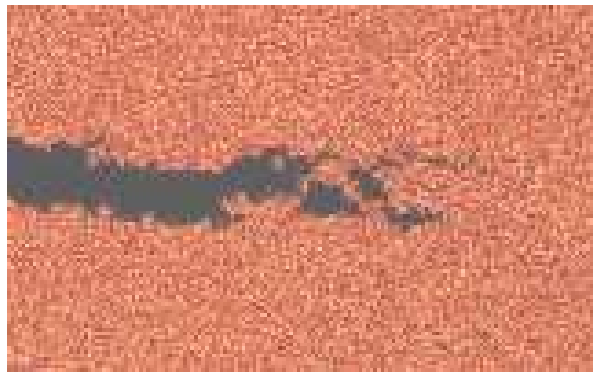


Echelle de longueur

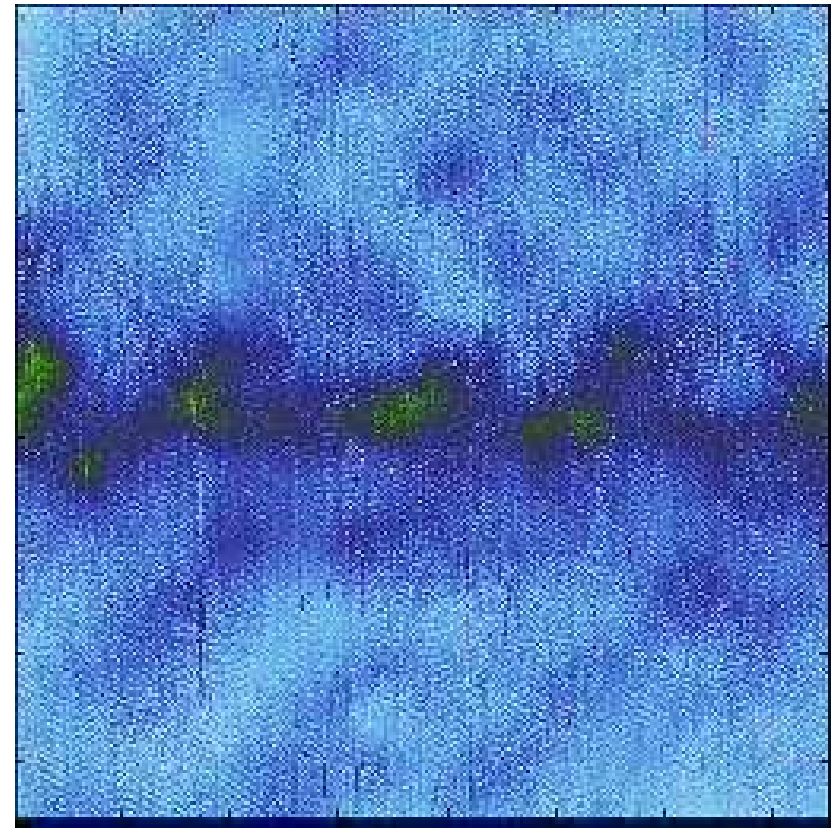


Aluminsosilicate en corrosion sous
contrainte, $V=10^{-10}$ m/s, (Celarie et al, 2003)

Silice, $V=300$ m/s, (Crédit
C. Rountree)



20 nm



75 nm

Fixée par l'endommagement?



PLAN DE L'EXPOSÉ

Mécanique de la rupture: Théorie **continue**

Rôle des défauts, failles de Griffith

Propagation de fissures: critère de stabilité,
équation de mouvement, trajectoire...

Rôle du désordre microstructural:
Description **stochastique**

Fissuration intermittente

ténacité effective

Faciès de rupture

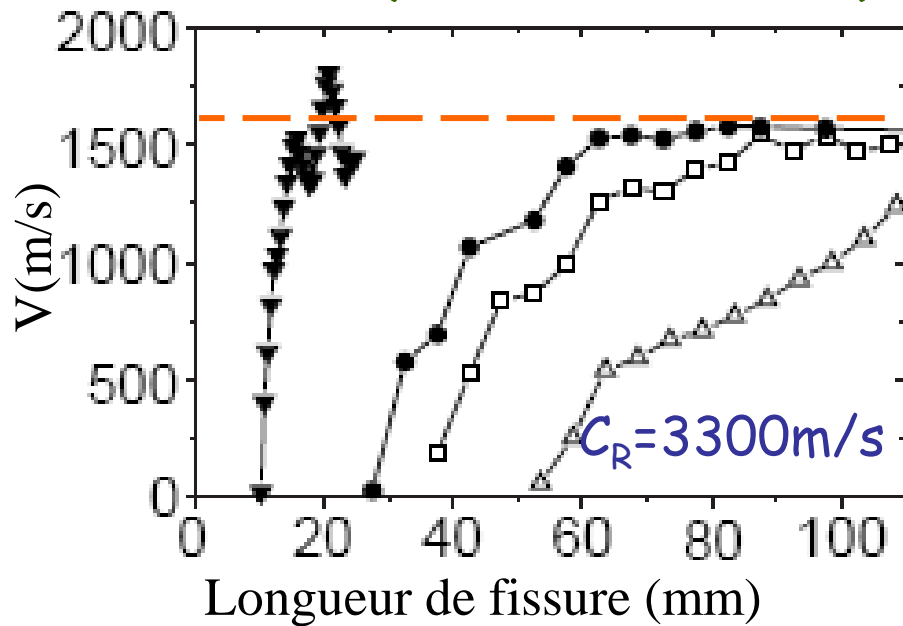
Instabilité dynamique, branchement



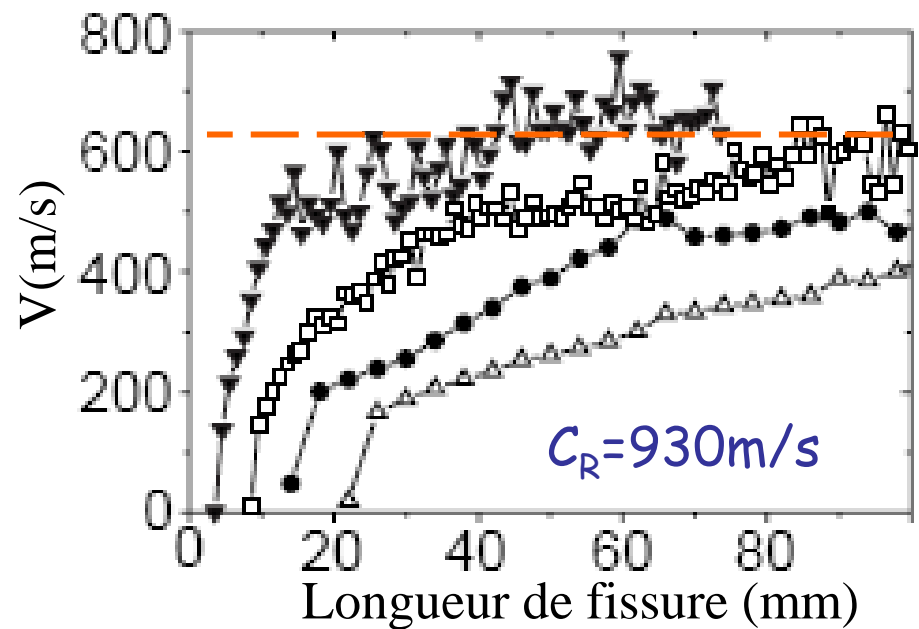
Equation de mouvement: prédictions

➤ Vitesse limite théorique = vitesse de Rayleigh

Verre (Sharon et al, 1999)



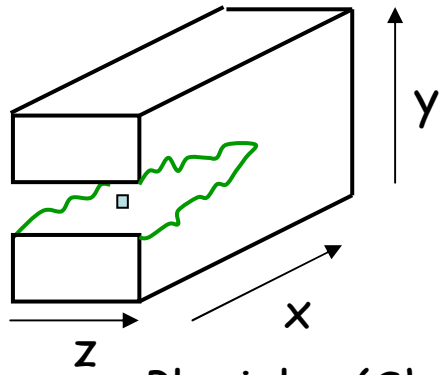
Plexiglas (Sharon et al, 1999)



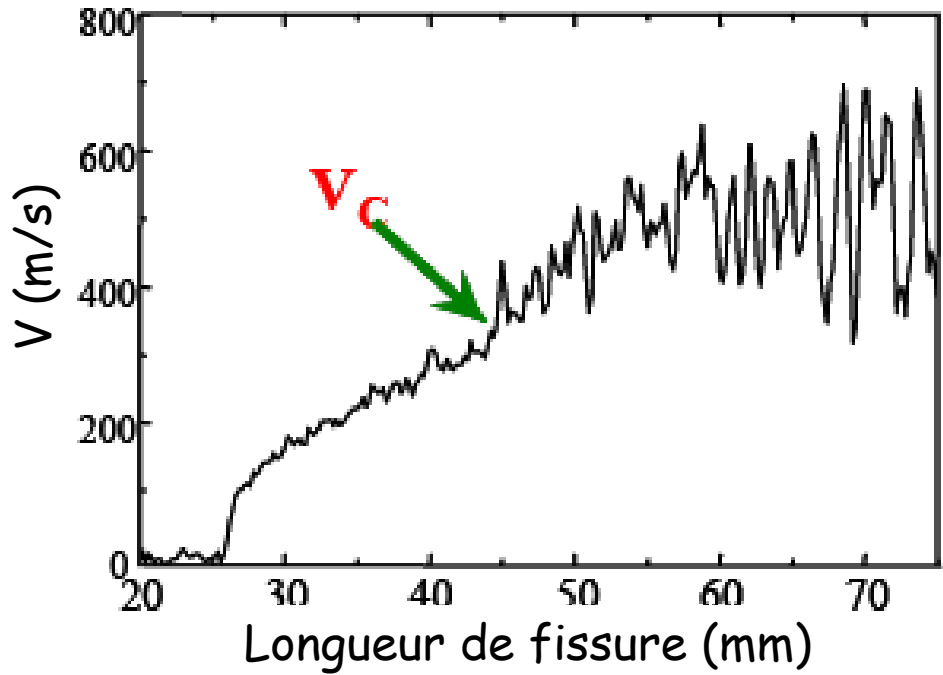
Expériences: $V_{\max} \ll \sim 0.6 C_R$!!!



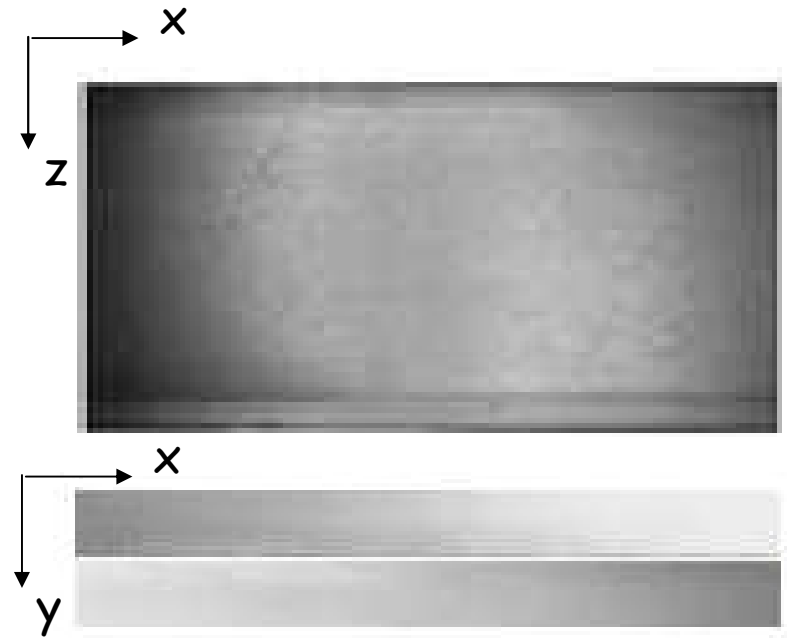
Rupture dynamique: instabilité de branchement



Plexiglas (Sharon et al, 1999)

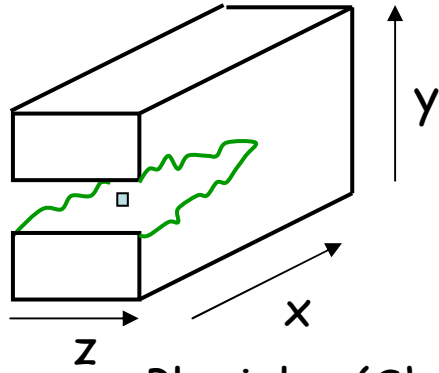


$$V < V_c$$

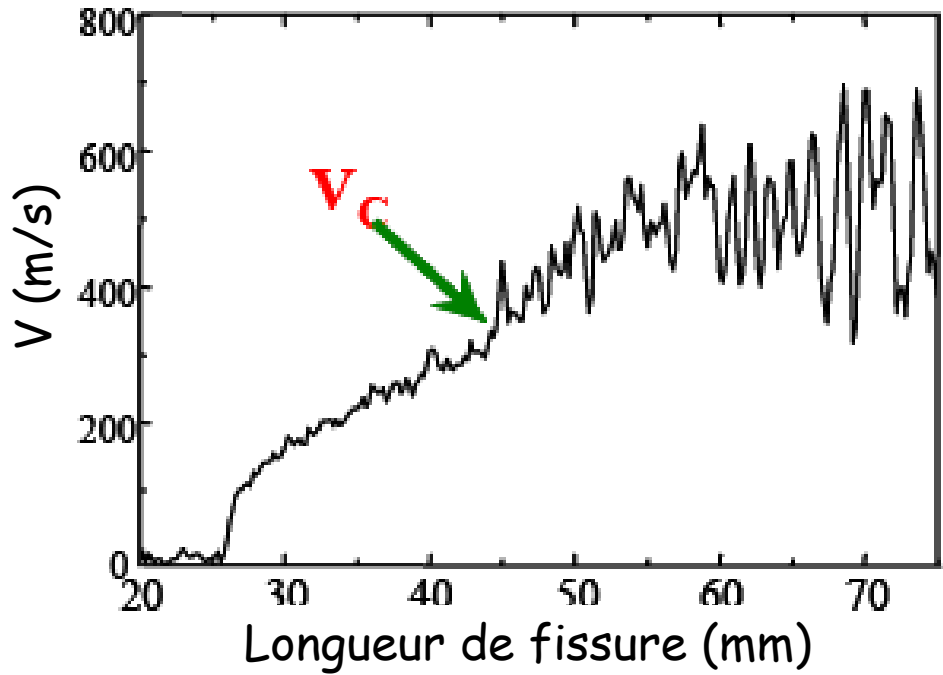




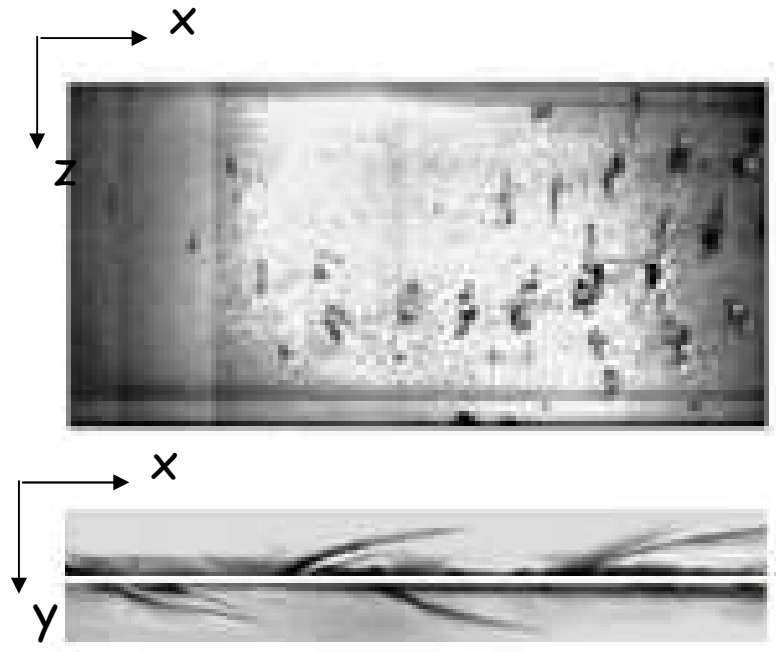
Rupture dynamique: instabilité de branchement



Plexiglas (Sharon et al, 1999)

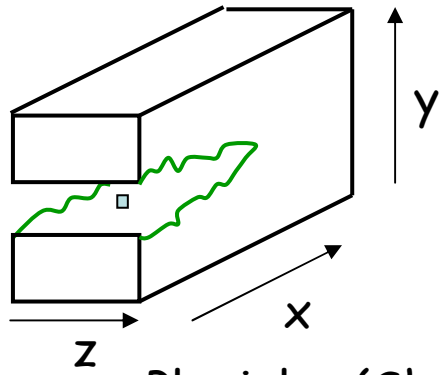


$$V \sim V_c$$

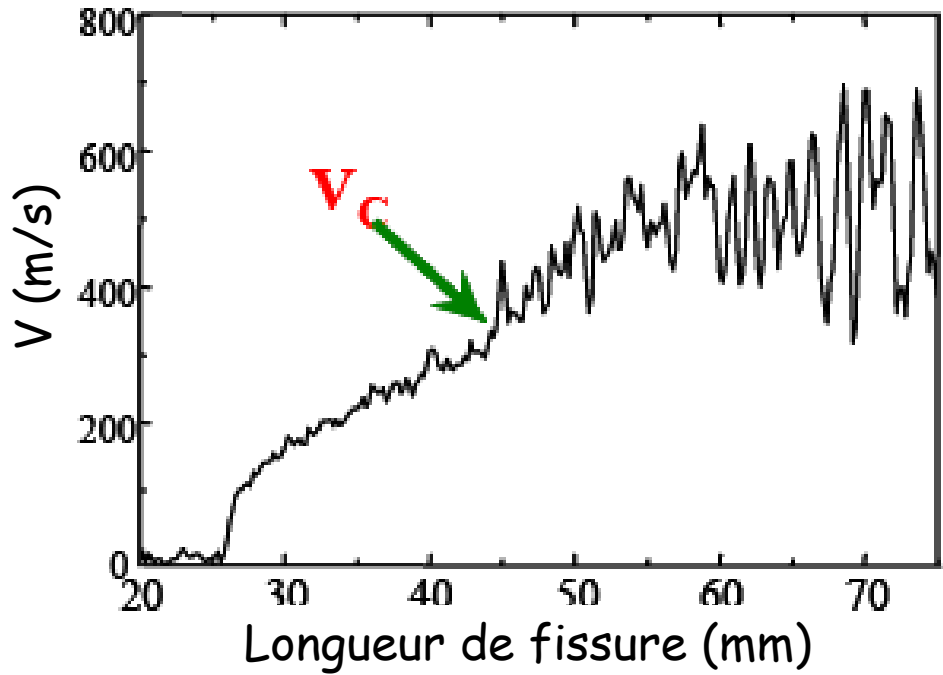




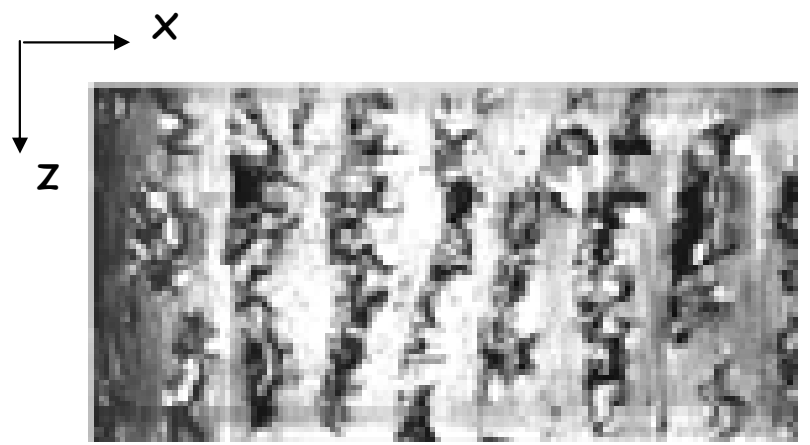
Rupture dynamique: instabilité de branchement



Plexiglas (Sharon et al, 1999)

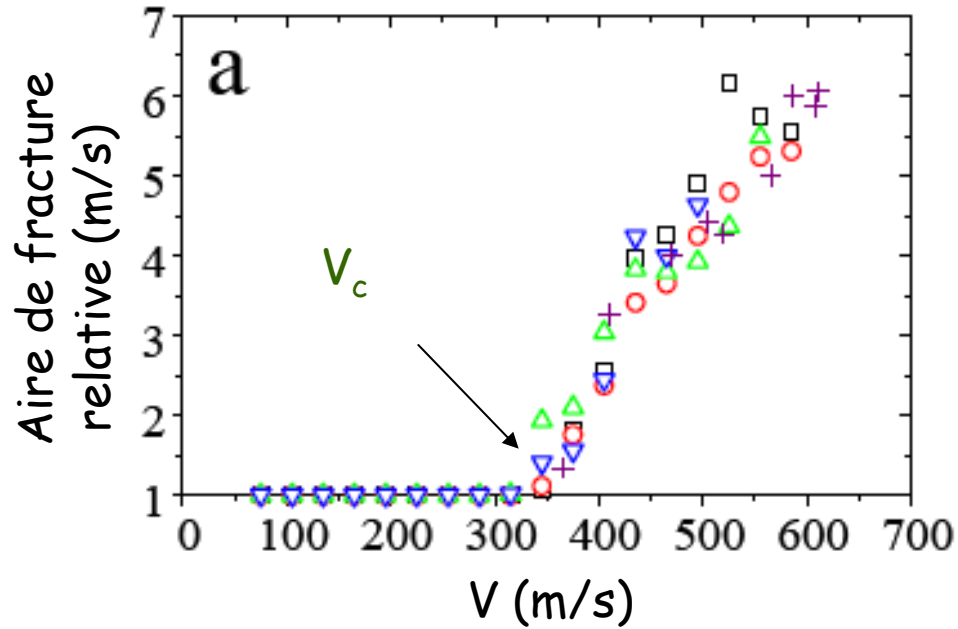


$V > V_c$

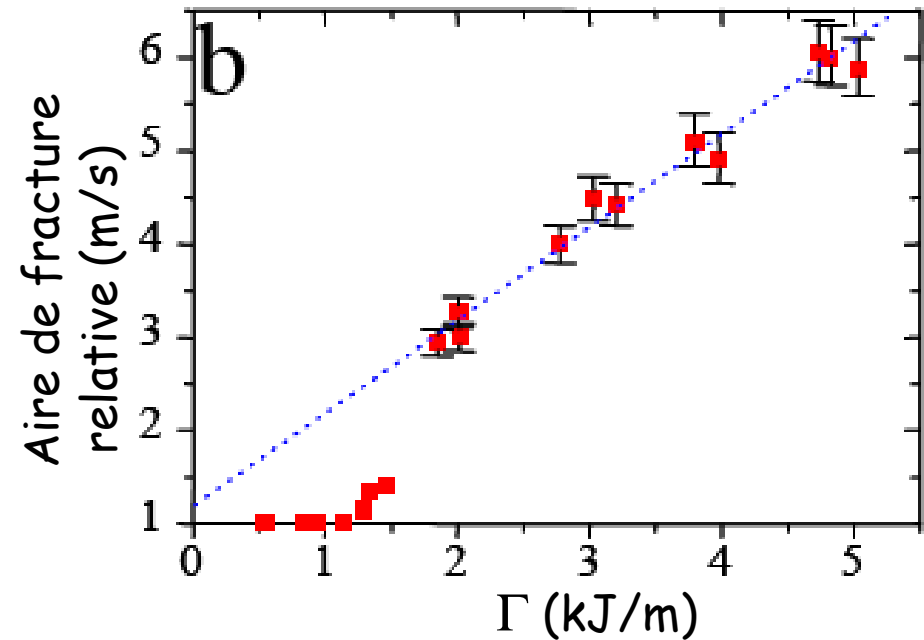




Rupture dynamique: instabilité de branchement



Plexiglas (Sharon et al, 1996)

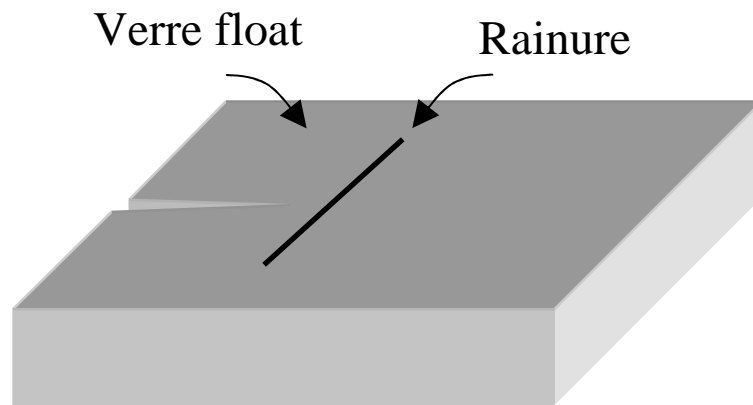


Une explication possible de $V_{\max} < C_R$



Rôle des hétérogénéités

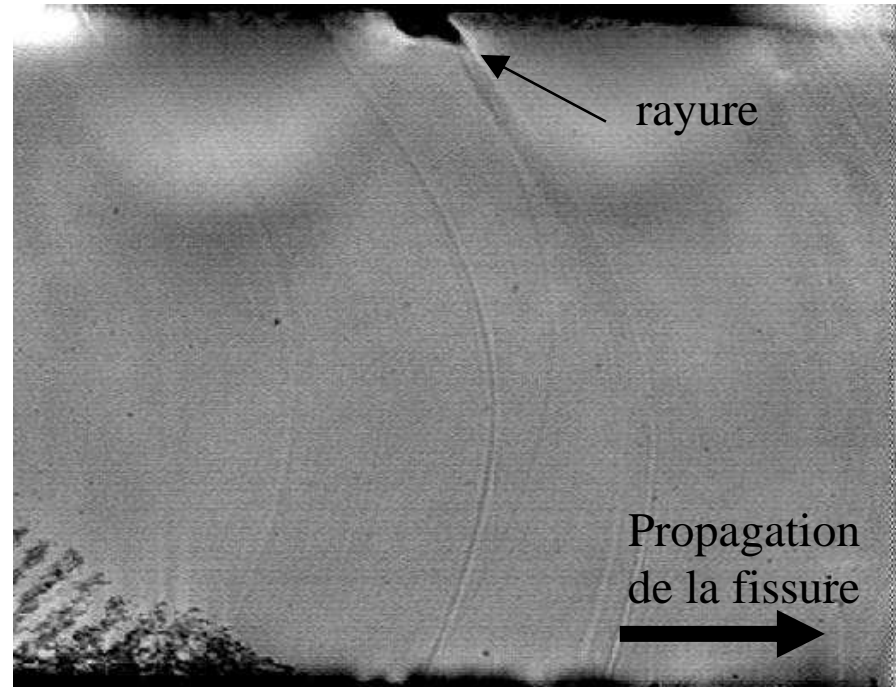
Wallner 1939, Sharon et al, 2001, DB & Ravi-Chandar 2003...



Propagation
de la fissure



$$0.1C_R < V < 0.4C_R$$

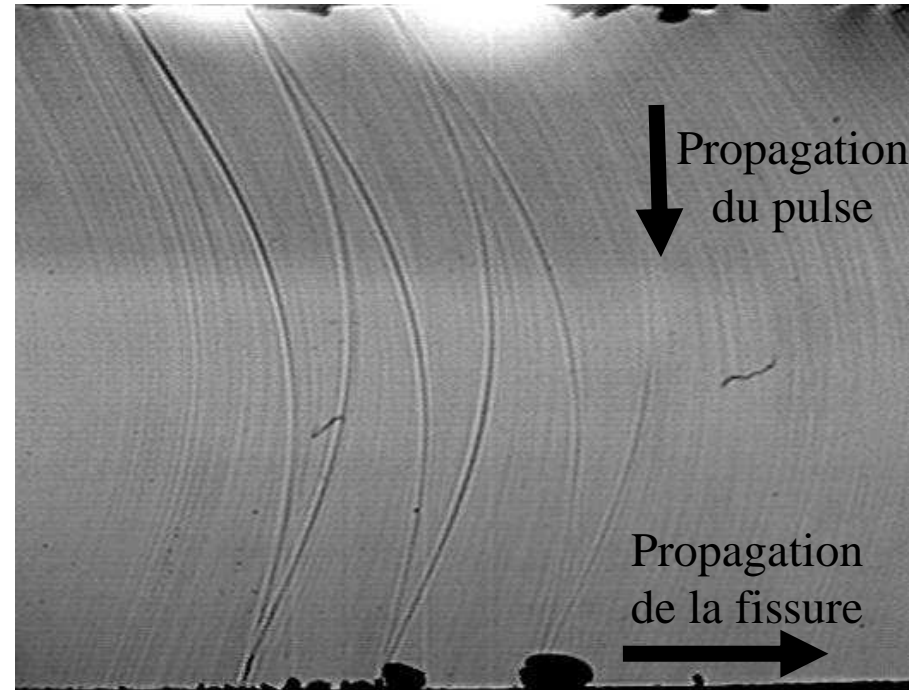
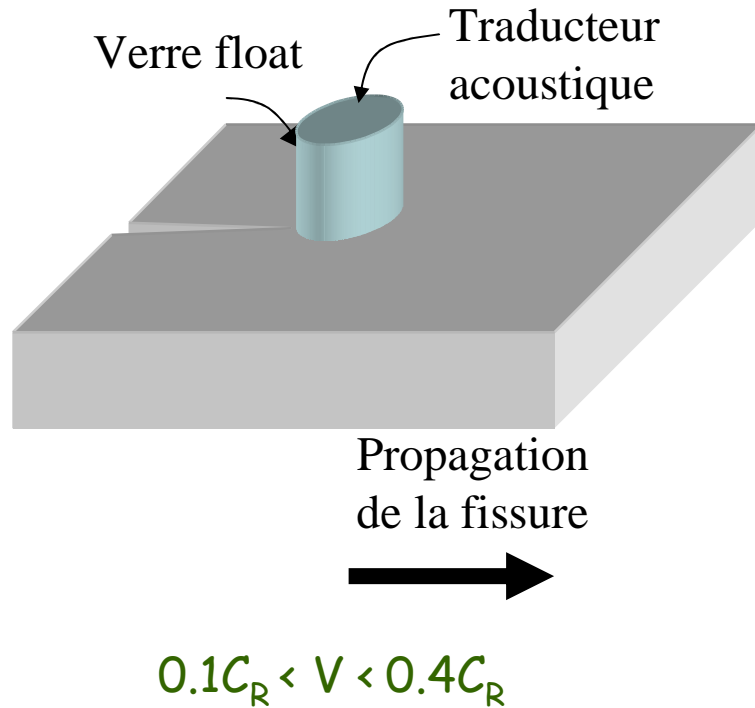


Faciès de rupture



Rôle des hétérogénéités

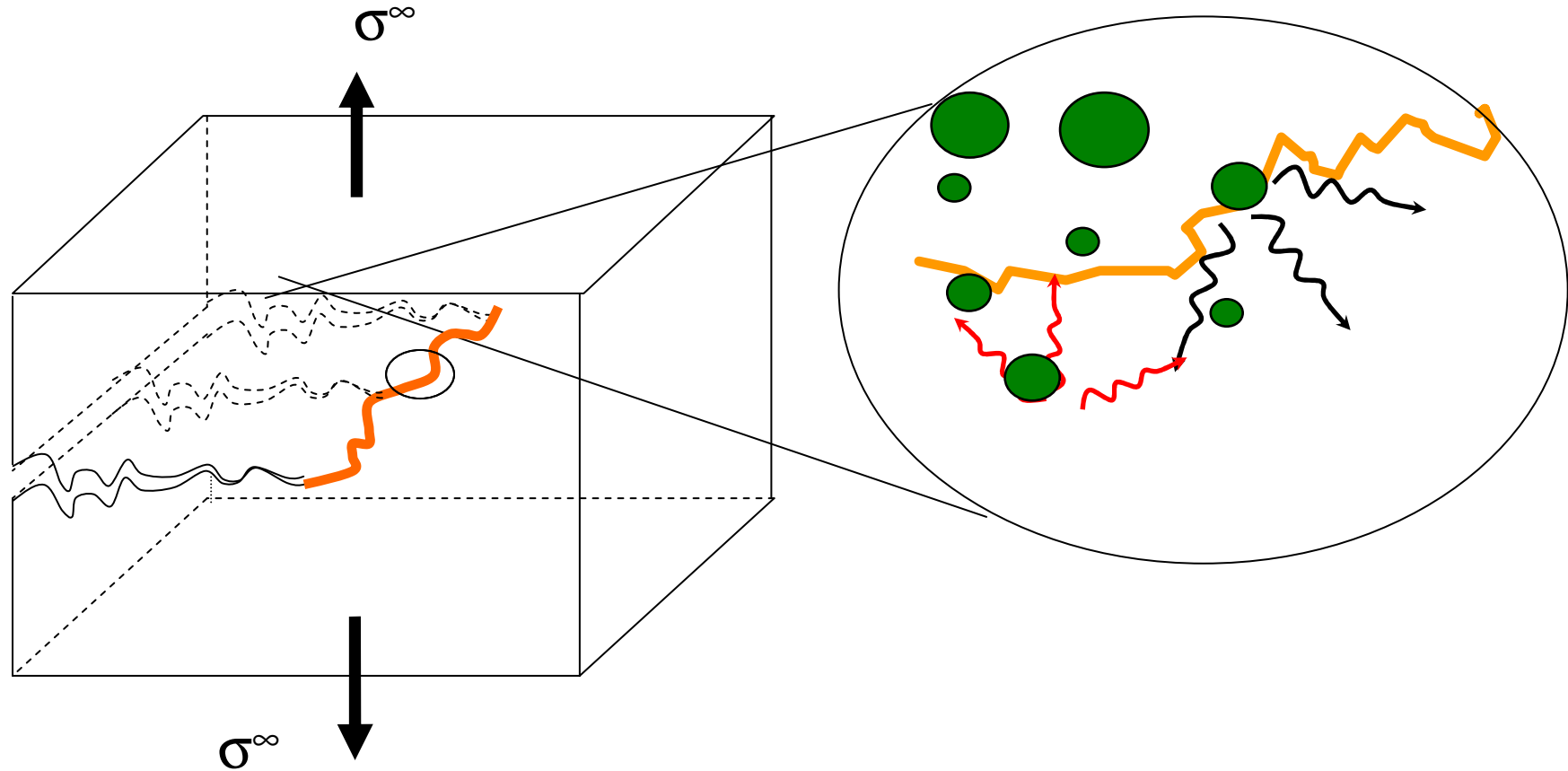
Wallner 1939, Sharon et al, 2001, DB & Ravi-Chandar 2003...



→ A haute vitesse, production de lignes de Wallner sans défaut/pulse acoustique introduit de manière externe



Instabilité de branchement... ... une origine possible





Conclusion

Certaines propriétés de rupture macroscopiques des verres semblent imposées par des fluctuations mécaniques ou structurales à très petites échelles.